Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 2, 2004*
Accepted: *April 12, 2004*

Two-loop QCD helicity amplitudes for massless quark-quark scattering

E. W. Nigel Glover

Department of Physics, University of Durham
Durham DH1 3LE, England
E-mail: E.W.N.Glover@durham.ac.uk

Abstract: We present the two-loop helicity amplitudes for the scattering of massless quarks in QCD. We use projector techniques to compute the coefficients of the general four-quark amplitude at up to two-loops in conventional dimensional regularisation and use these coefficients to derive the helicity amplitudes in the 't Hooft-Veltman scheme. The structure of the infrared divergences agrees with that predicted by Catani while expressions for the finite remainders for $q\bar{q} \to Q\bar{Q}$ and the processes related by crossing symmetry are given in terms of logarithms and polylogarithms that are real in the physical region. We have checked that the interference of tree and two-loop helicity amplitudes, summed over helicities and colours, reproduces the previous results for the finite remainders for interference of tree and two-loop amplitudes given in ref. [1, 2].

Keywords: QCD, NLO Computations, Jets.

Contents

1. Introduction

In the past decade, QCD has become a quantitative science and comparisons of high quality experimental data from high energy collider experiments with QCD predictions at next-to-leading order (NLO) in α_s have become the *de facto* standard. In such studies, the parton level predictions serve to provide an estimate of the uncalculated theoretical uncertainties via the variation in the renormalisation (and factorisation) scale. However, there are many well established reasons why extending perturbative calculations to next-to-next-to-leading order (NNLO) is vital in reducing the theoretical uncertainty for the dominant $2 \to 1$ and $2 \to 2$ processes such as Drell-Yan or jet production. Most of the anticipated improvements are either due (a) to including the next term in the perturbation series or (b) to including the effect of more partons in the final state. Here, we list six of the improvements expected for NNLO predictions at hadron colliders and refer the reader to ref. [3] for a more detailed discussion;

1. (a) Reduction of the renormalisation scale uncertainty;
2. (a) Reduction of the factorisation scale uncertainty;
3. (a) Reduced power corrections;
4. (b) Better matching between parton-level and hadron level jet algorithms;
5. (b) Better description of the transverse momentum of the initial state;
6. (b) Extended kinematic coverage due to enlarged phase space.

All of these will be underpinned by a better knowledge of the parton distributions using global fits to NNLO observables from DIS, Drell-Yan and jet production and by a better determination of α_s obtained from 3 jet event shape data at electron-positron colliders. Taken together, one expects that the theoretical uncertainty on a given observable will drop from $\mathcal{O}(20\%)$ at NLO to $< \mathcal{O}(10\%)$ at NNLO.

In this paper, we focus on jet production in hadron-hadron collisions. The full NNLO prediction requires a knowledge of the two-loop $2 \to 2$ partonic matrix elements, as well as the one-loop $2 \to 3$ and tree-level $2 \to 4$ amplitudes and NNLO parton distributions. Much work has been invested in the three-loop splitting functions [4]–[14] needed to evolve the parton distribution functions at this order. The available moments have been used to provide approximate fits in x-space and NNLO parton distributions are beginning to become available [15, 16]. The exact three-loop splitting functions have recently been computed [17, 18]. On the other hand, the tree-level six-point amplitudes [19]–[26] and one-loop five point amplitudes [27, 28, 29] have been known for some time while the interference of tree and two-loop amplitudes [1, 2, 30, 31] and self-interference of the one-loop amplitudes [30, 32, 33] are also available. Subsequently, two-loop helicity amplitudes have been evaluated for the gluon-gluon [34, 35] and quark-gluon [36, 37] processes and have confirmed the earlier "squared" matrix elements.[1] Here, we complete the set of two-loop QCD helicity amplitudes for parton-parton scattering by studying processes involving two pairs of massless quarks,

$$q\bar{q} \to Q\bar{Q}\,,$$

and the processes related by time reversal and crossing symmetry when q and Q are either distinct or identical.

We extract the helicity amplitudes using the same method as that employed in refs. [41, 39, 37]. First we write down a general tensor that describes the four quark amplitude. Because the Dirac algebra is infinite dimensional for non-integer d, we use a basis set of Dirac strings that are sufficient to describe the amplitude up to two-loops. At three-loops new structures will appear. The coefficients are then extracted using a set of projectors that are valid at up to two-loops. These projectors turn the Dirac strings into traces (which are taken in d-dimensions) and thereby reduce the problem to evaluating scalar integrals. The methodology of reducing these scalar integrals to master integrals is exactly the same as for the calculation of the spin summed two-loop matrix elements [1, 2]. By taking the trace in d-dimensions and keeping all Lorentz indices in d-dimensions, we ensure that tensor coefficients are evaluated in conventional dimensional regularisation (CDR). We then provide the perturbative expansion for the one- and two-loop tensor coefficients and remove ultraviolet (UV) divergences at each order in α_s by renormalisation within the $\overline{\text{MS}}$ scheme. The infrared (IR) divergent structure is shown to agree with that obtained in ref. [42]. Finally, the helicity amplitudes are obtained from the general tensor using standard helicity techniques and within the t'Hooft-Veltman (HV) scheme where the external states are four-dimensional and the internal states are kept in $d = 4 - 2\epsilon$-dimensions [43, 44]. The finite helicity amplitudes are the main new results presented in this paper and we give explicit analytic expressions valid for each helicity configuration and process in terms of logarithms and polylogarithms that are real in the physical domain.

The paper is organised as follows. In section 2 we introduce notation and discuss the general tensor structure of the four-quark amplitude at up to two-loops. We give projectors to isolate the various coefficients and relate the individual coefficients to the interference of amplitudes. Section 3 shows the perturbative expansion for the tensor coefficients computed within CDR and presents an analysis of their UV and IR divergent structure. The coefficients are renormalised in the $\overline{\text{MS}}$ scheme to remove all UV divergences, while the IR poles are predicted using the results of ref. [42]. Expressions for the one-loop tensor coefficients to all orders in ϵ appear in section 3.2 in terms of the one-loop bubble integral and the one-loop box integral in $d = 6 - 2\epsilon$. The relationship between the tensor coefficients and the helicity amplitudes in the HV scheme is described in section 4. The

[1]Note that helicity amplitudes for the phenomenologically important $gg \to \gamma\gamma$ process [38], $e^+e^- \to$ 3 jets [39] and $\gamma\gamma \to \gamma\gamma$ [40, 41] have also been computed.

JHEP04(2004)021

UV and IR structure of the helicity amplitudes is exactly the same as for the tensor coefficients, and finite $\mathcal{O}\left(\epsilon^0\right)$ remainders are obtained by subtracting the predicted pole structure from the explicit calculation both at one-loop and two-loop. The main results of this paper are the finite remainders of the two-loop helicity amplitudes. The expressions are rather lengthy and we present the amplitudes for the four-quark process in appendix A. For completeness, the finite one-loop remainders are collected in appendix B. Finally, in section 6 we summarize our results.

2. Notation

The processes that we wish to consider in detail are,

$$0 \to q\bar{q}Q\bar{Q}\,, \tag{2.1}$$

$$0 \to q\bar{q}q\bar{q}\,. \tag{2.2}$$

Because the identical quark amplitudes are obtained from the non-identical quark amplitudes by the exchange of the quark or antiquark momenta, we focus on the scattering of distinct quarks,

$$0 \to q(p_1,\lambda_1) + \bar{q}(p_2,\lambda_2) + Q(p_3,\lambda_3) + \bar{Q}(p_4,\lambda_4)\,, \tag{2.3}$$

where the quarks q and Q are taken to be massless with momenta satisfying,

$$0 \to p_1^\mu + p_2^\mu + p_3^\mu + p_4^\mu\,, \qquad p_i^2 = 0\,. \tag{2.4}$$

Physical processes are obtained by crossing particles into the initial state. The associated Mandelstam variables are given by

$$s_{12} = (p_1+p_2)^2\,, \qquad s_{23} = (p_2+p_3)^2\,, \qquad s_{13} = (p_1+p_3)^2\,, \qquad s_{12}+s_{23}+s_{13} = 0\,. \tag{2.5}$$

We work in conventional dimensional regularisation and renormalise the ultraviolet divergences in the $\overline{\rm MS}$ scheme. The bare coupling α_0 is related to the running coupling $\alpha_s \equiv \alpha_s(\mu^2)$ at renormalisation scale μ, by

$$\alpha_0\mu_0^{2\epsilon}\, S_\epsilon = \alpha_s\,\mu^{2\epsilon}\left[1 - \frac{\beta_0}{\epsilon}\left(\frac{\alpha_s}{2\pi}\right) + \left(\frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1}{2\epsilon}\right)\left(\frac{\alpha_s}{2\pi}\right)^2 + \mathcal{O}\left(\alpha_s^3\right)\right], \tag{2.6}$$

where

$$S_\epsilon = (4\pi)^\epsilon e^{-\epsilon\gamma}\,, \qquad \gamma = 0.5772\ldots = \text{Euler constant} \tag{2.7}$$

is the typical phase-space volume factor in $d = 4-2\epsilon$ dimensions and μ_0^2 is the mass parameter introduced in dimensional regularisation [45, 46, 43, 44] to maintain a dimensionless coupling in the bare QCD lagrangian density.

The first two coefficients of the QCD beta function, β_0 and β_1, for N_F (massless) quark flavours, are

$$\beta_0 = \frac{11C_A - 4T_RN_F}{6}\,, \qquad \beta_1 = \frac{17C_A^2 - 10C_AT_RN_F - 6C_FT_RN_F}{6}\,, \tag{2.8}$$

where N is the number of colours and

$$C_F = \frac{N^2-1}{2N}\,, \qquad C_A = N\,, \qquad T_R = \frac{1}{2}\,. \tag{2.9}$$

JHEP04(2004)021

2.1 The general tensor

The most general tensor structure for the amplitude, $|\mathcal{M}\rangle$, for the scattering of distinct massless quarks at up to two loops can be written as

$$|\mathcal{M}\rangle = \sum_{I=1}^{6} A_I(s_{12}, s_{23})\ \mathcal{D}_I + \cdots \tag{2.10}$$

where the coefficients A_I are vectors in colour space and are functions of s_{12} and s_{23} (and implicitly $s_{13} = -s_{12} - s_{23}$) where $s_{ij} = (p_i + p_j)^2$ and the six Dirac structures are

$$\begin{aligned}
\mathcal{D}_1 &= \bar{u}(p_1)\gamma_{\mu_1}u(p_2)\ \bar{u}(p_3)\gamma_{\mu_1}u(p_4)\,,\\
\mathcal{D}_2 &= \bar{u}(p_1)\not{p}_3 u(p_2)\ \bar{u}(p_3)\not{p}_1 u(p_4)\,,\\
\mathcal{D}_3 &= \bar{u}(p_1)\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}u(p_2)\ \bar{u}(p_3)\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}u(p_4)\,,\\
\mathcal{D}_4 &= \bar{u}(p_1)\gamma_{\mu_1}\not{p}_3\gamma_{\mu_3}u(p_2)\ \bar{u}(p_3)\gamma_{\mu_1}\not{p}_1\gamma_{\mu_3}u(p_4)\,,\\
\mathcal{D}_5 &= \bar{u}(p_1)\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}u(p_2)\bar{u}(p_3)\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}u(p_4)\,,\\
\mathcal{D}_6 &= \bar{u}(p_1)\gamma_{\mu_1}\gamma_{\mu_2}\not{p}_3\gamma_{\mu_4}\gamma_{\mu_5}u(p_2)\ \bar{u}(p_3)\gamma_{\mu_1}\gamma_{\mu_2}\not{p}_1\gamma_{\mu_4}\gamma_{\mu_5}u(p_4)\,.
\end{aligned} \tag{2.11}$$

This tensor structure is a priori d-dimensional since the Lorentz indices are d-dimensional and the dimensionality (and helicity) of the external states has not yet been specified. One can in principle relate the strings of gamma matrices appearing in $\mathcal{D}_3$ to $\mathcal{D}_6$ to a standard set involving only $\mathcal{D}_1$ and $\mathcal{D}_2$ using four-dimensional tricks. However, because these are the structures that naturally arise in the parity conserving interactions of QCD, we choose to use this extended set as a d-dimensional basis that is valid at up to two-loops. We note that the Dirac algebra is infinite dimensional for non-integer d and that the basis set will extend according to the order that $|\mathcal{M}\rangle$ is computed. For example, at tree level, only $\mathcal{D}_1$ appears, while $\mathcal{D}_2$, $\mathcal{D}_3$ and $\mathcal{D}_4$ first appear at one-loop. $\mathcal{D}_5$ and $\mathcal{D}_6$ appear for the first time at two-loops while at three-loops, we will find terms (represented by $+\cdots$) with seven gamma matrices sandwiched between the quark spinors. These more complicated structures can also be related to the simpler ones using four-dimensional tricks (which we choose not to do at the present time).

When the quarks are identical, the general structure of the amplitude is modified,

$$|M\rangle = |\mathcal{M}\rangle - \delta_{qQ}|\overline{\mathcal{M}}\rangle\,, \tag{2.12}$$

where

$$|\overline{\mathcal{M}}\rangle = |\mathcal{M}\rangle(p_2 \leftrightarrow p_4)\,. \tag{2.13}$$

The minus sign is due to the exchange of identical fermions, while the momentum swap corresponds to exchanging s_{12} and s_{23} in the coefficents A_I. All appropriate colour indices are also exchanged. In general we will multiply these additional identical fermion terms with a δ_{qQ} which is unity when the quarks are identical and zero otherwise.

JHEP04(2004)021

2.2 Projectors for the tensor coefficients

The six coefficients A_I may be easily extracted from a Feynman diagram calculation with two distinct quark flavours using projectors that act on the general tensor of eq. (2.10) such that

$$\sum_{\text{spins}} \mathcal{P}(A_I)\ |\mathcal{M}\rangle = A_I(s_{12}, s_{23})\,. \tag{2.14}$$

The explicit forms for the projectors in d space-time dimensions are,

$$\begin{aligned}
\mathcal{P}(A_1) = {}& \frac{1}{480 s_{13} s_{23}^2 s_{12}^2 (d-5)(d-6)(d-7)(d-3)(d-4)} \times \\
& \times\Big(+ s_{13}(-9240 s_{23}^2 d^3 - 35040 s_{13} s_{23} d^2 + 52160 s_{13}^2 d - 61120 s_{13}^2 + 61620 s_{23}^2 d^2 + \\
& \qquad + 164320 s_{13} s_{23} d + 202496 s_{23}^2 - 12720 s_{13}^2 d^2 - 182480 s_{23}^2 d + \\
& \qquad + 960 s_{13}^2 d^3 - 259840 s_{13} s_{23} + 525 s_{23}^2 d^4 + 2520 s_{13} s_{23} d^3)\mathcal{D}_1^\dagger - \\
& \quad - 10 s_{13}(24 s_{13}^2 d^2 - 952 s_{13} s_{23} d + 102 s_{13} s_{23} d^2 - 1568 s_{23}^2 + 2344 s_{13} s_{23} - \\
& \qquad - 264 s_{23}^2 d^2 + 21 s_{23}^2 d^3 + 256 s_{13}^2 - 176 s_{13}^2 d + 1124 s_{23}^2 d)\mathcal{D}_3^\dagger - \\
& \quad - 15(d-6)(35 s_{23}^2 d^3 - 55 s_{13} s_{23} d^3 + 1046 s_{13} s_{23} d^2 - 1872 s_{13}^2 d + \\
& \qquad + 2432 s_{13}^2 - 454 s_{23}^2 d^2 - 6040 s_{13} s_{23} d - 2688 s_{23}^2 + \\
& \qquad + 368 s_{13}^2 d^2 + 1928 s_{23}^2 d - 20 s_{13}^2 d^3 + 11136 s_{13} s_{23})\mathcal{D}_2^\dagger + \\
& \quad + s_{13} s_{23}(-320 s_{13} + 15 s_{23} d^2 - 110 s_{23} d + 224 s_{23} + 60 s_{13} d)\mathcal{D}_5^\dagger - \\
& \quad - 5(-102 s_{23}^2 d + 15 s_{23}^2 d^2 - 1048 s_{13} s_{23} + 168 s_{23}^2 + 88 s_{13}^2 d - \\
& \qquad - 128 s_{13}^2 - 27 s_{13} s_{23} d^2 + 326 s_{13} s_{23} d - 12 s_{13}^2 d^2)\mathcal{D}_6^\dagger \\
& \quad + 30(21 s_{23}^2 d^3 - 37 s_{13} s_{23} d^3 + 672 s_{13} s_{23} d^2 - 1104 s_{13}^2 d + 1360 s_{13}^2 - \\
& \qquad - 256 s_{23}^2 d^2 - 3868 s_{13} s_{23} d - 1344 s_{23}^2 + 244 s_{13}^2 d^2 + 1036 s_{23}^2 d - \\
& \qquad - 16 s_{13}^2 d^3 + 7328 s_{13} s_{23})\mathcal{D}_4^\dagger\Big)\,, \\
\mathcal{P}(A_2) = {}& \frac{1}{32 s_{13}^2 s_{23}^2 s_{12}^2 (d-5)(d-7)(d-3)(d-4)} \times \\
& \times\Big(-s_{13}(35 s_{23}^2 d^3 - 55 s_{13} s_{23} d^3 + 1046 s_{13} s_{23} d^2 - 1872 s_{13}^2 d + 2432 s_{13}^2 - \\
& \qquad - 454 s_{23}^2 d^2 - 6040 s_{13} s_{23} d - 2688 s_{23}^2 + 368 s_{13}^2 d^2 + 1928 s_{23}^2 d - \\
& \qquad - 20 s_{13}^2 d^3 + 11136 s_{13} s_{23})\mathcal{D}_1^\dagger + \\
& \quad + 2 s_{13}(-2 s_{13}^2 d^2 - 9 s_{13} s_{23} d^2 + 142 s_{13} s_{23} d - 448 s_{13} s_{23} + 7 s_{23}^2 d^2 + \\
& \qquad + 136 s_{23}^2 - 48 s_{13}^2 + 28 s_{13}^2 d - 62 s_{23}^2 d)\mathcal{D}_3^\dagger + \\
& \quad +(-340 s_{13}^2 d^3 + 11008 s_{13}^2 - 740 s_{13} s_{23} d^3 + 44032 s_{13} s_{23} - 260 s_{23}^2 d^3 - \\
& \qquad - 4144 s_{23}^2 d + 3712 s_{23}^2 + 15 s_{13}^2 d^4 + 2852 s_{13}^2 d^2 - 28864 s_{13} s_{23} d + 1604 s_{23}^2 d^2 + \\
& \qquad + 6944 s_{13} s_{23} d^2 - 9968 s_{13}^2 d + 30 s_{13} s_{23} d^4 + 15 s_{23}^2 d^4)\mathcal{D}_2^\dagger - \\
& \quad - s_{13} s_{23}(12 s_{13} + s_{23} d - 4 s_{23} - s_{13} d)\mathcal{D}_5^\dagger + \\
& \quad +(-6 s_{23}^2 d + 24 s_{13}^2 + 2 s_{13} s_{23} d^2 - 40 s_{13} s_{23} d - 14 s_{13}^2 d + s_{13}^2 d^2 + \\
& \qquad + 8 s_{23}^2 + s_{23}^2 d^2 + 192 s_{13} s_{23})\mathcal{D}_6^\dagger -
\end{aligned}$$

JHEP04(2004)021

$$
\begin{aligned}
&- 2(5s_{13}^2d^3 + 5s_{23}^2d^3 + 10s_{13}s_{23}d^3 - 240s_{13}s_{23}d^2 - 100s_{13}^2d^2 - \\
&- 56s_{23}^2d^2 + 580s_{13}^2d + 1832s_{13}s_{23}d + 196s_{23}^2d - 208s_{23}^2 - \\
&- 800s_{13}^2 - 4224s_{13}s_{23})\mathcal{D}_4^\dagger\Big)\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}(A_3) = & \frac{1}{48s_{13}s_{23}^2s_{12}^2(d-5)(d-6)(d-7)(d-3)(d-4)} \times \\
& \times\Big(-s_{13}(24s_{13}^2d^2 - 952s_{13}s_{23}d + 102s_{13}s_{23}d^2 - 1568s_{23}^2 + 2344s_{13}s_{23} - \\
&\quad - 264s_{23}^2d^2 + 21s_{23}^2d^3 + 256s_{13}^2 - 176s_{13}^2d + 1124s_{23}^2d)\mathcal{D}_1^\dagger + \\
&\quad + 2s_{13}(20s_{13}s_{23}d + 122s_{23}^2 + 4s_{13}^2d - 8s_{13}^2 + 6s_{23}^2d^2 - 53s_{23}^2d - 80s_{13}s_{23})\mathcal{D}_3^\dagger + \\
&\quad + 3(d-6)(-2s_{13}^2d^2 - 9s_{13}s_{23}d^2 + 142s_{13}s_{23}d - 448s_{13}s_{23} + 7s_{23}^2d^2 + \\
&\qquad\quad + 136s_{23}^2 - 48s_{13}^2 + 28s_{13}^2d - 62s_{23}^2d)\mathcal{D}_2^\dagger - \\
&\quad - s_{13}s_{23}(s_{23}d - 4s_{23} + 2s_{13})\mathcal{D}_5^\dagger + \\
&\quad + (-20s_{23}^2 - 7s_{13}s_{23}d + 5s_{23}^2d + 4s_{13}^2 - 2s_{13}^2d + 44s_{13}s_{23})\mathcal{D}_6^\dagger - \\
&\quad - 6(-2s_{13}^2d^2 - 8s_{13}s_{23}d^2 + 105s_{13}s_{23}d - 298s_{13}s_{23} + 6s_{23}^2d^2 + 122s_{23}^2 - \\
&\quad - 32s_{13}^2 + 20s_{13}^2d - 53s_{23}^2d)\mathcal{D}_4^\dagger
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}(A_4) = & \frac{1}{16s_{13}^2s_{23}^2s_{12}^2(d-5)(d-6)(d-7)(d-3)(d-4)} \times \\
& \times\Big(+ s_{13}(21s_{23}^2d^3 - 37s_{13}s_{23}d^3 + 672s_{13}s_{23}d^2 - 1104s_{13}^2d + 1360s_{13}^2 - \\
&\qquad - 256s_{23}^2d^2 - 3868s_{13}s_{23}d - 1344s_{23}^2 + 244s_{13}^2d^2 + 1036s_{23}^2d - \\
&\qquad - 16s_{13}^2d^3 + 7328s_{13}s_{23})\mathcal{D}_1^\dagger - \\
&\quad - 2s_{13}(-2s_{13}^2d^2 - 8s_{13}s_{23}d^2 + 105s_{13}s_{23}d - 298s_{13}s_{23} + 6s_{23}^2d^2 + \\
&\qquad + 122s_{23}^2 - 32s_{13}^2 + 20s_{13}^2d - 53s_{23}^2d)\mathcal{D}_3^\dagger - \\
&\quad - (d-6)(5s_{13}^2d^3 + 5s_{23}^2d^3 + 10s_{13}s_{23}d^3 - 240s_{13}s_{23}d^2 - 100s_{13}^2d^2 - \\
&\qquad\quad - 56s_{23}^2d^2 + 580s_{13}^2d + 1832s_{13}s_{23}d + 196s_{23}^2d - 208s_{23}^2 - \\
&\qquad\quad - 800s_{13}^2 - 4224s_{13}s_{23})\mathcal{D}_2^\dagger \\
&\quad + s_{13}s_{23}(-s_{13}d - 4s_{23} + s_{23}d + 8s_{13})\mathcal{D}_5^\dagger - \\
&\quad - (16s_{13}^2 + 144s_{13}s_{23} - 36s_{13}s_{23}d + 2s_{13}s_{23}d^2 - 10s_{13}^2d \\
&\qquad - 6s_{23}^2d + 8s_{23}^2 + s_{23}^2d^2 + s_{13}^2d^2)\mathcal{D}_6^\dagger + \\
&\quad + 2(-186s_{13}s_{23}d^2 + 4s_{13}^2d^3 + 356s_{13}^2d - 69s_{13}^2d^2 - 2976s_{13}s_{23} - 468s_{13}^2 - \\
&\qquad - 180s_{23}^2 + 8s_{13}s_{23}d^3 - 45s_{23}^2d^2 + 1348s_{13}s_{23}d + 164s_{23}^2d + 4s_{23}^2d^3)\mathcal{D}_4^\dagger\Big)\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}(A_5) = & \frac{1}{480s_{13}s_{23}s_{12}^2(d-5)(d-6)(d-7)(d-3)(d-4)} \times \\
& \times\Big(+ s_{13}(-320s_{13} + 15s_{23}d^2 - 110s_{23}d + 224s_{23} + 60s_{13}d)\mathcal{D}_1^\dagger - \\
&\quad - 10s_{13}(s_{23}d - 4s_{23} + 2s_{13})\mathcal{D}_3^\dagger - 15(d-6)(12s_{13} + s_{23}d - 4s_{23} - s_{13}d)\mathcal{D}_2^\dagger \\
&\quad + s_{13}s_{23}\mathcal{D}_5^\dagger - 5(-s_{13} + s_{23})\mathcal{D}_6^\dagger 30(-s_{13}d - 4s_{23} + s_{23}d + 8s_{13})\mathcal{D}_4^\dagger\Big)\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}(A_6) =& \frac{1}{96 s_{13}^2 s_{23}^2 s_{12}^2 (d-5)(d-6)(d-7)(d-3)(d-4)} \times \\
&\times\Big(- s_{13}(-102 s_{23}^2 d + 15 s_{23}^2 d^2 - 1048 s_{13} s_{23} + 168 s_{23}^2 + 88 s_{13}^2 d - \\
&\qquad - 128 s_{13}^2 - 27 s_{13} s_{23} d^2 + +326 s_{13} s_{23} d - 12 s_{13}^2 d^2)\mathcal{D}_1^\dagger + \\
&\quad + 2 s_{13}(-20 s_{23}^2 - 7 s_{13} s_{23} d + 5 s_{23}^2 d + 4 s_{13}^2 - 2 s_{13}^2 d + 44 s_{13} s_{23})\mathcal{D}_3^\dagger + \\
&\quad + 3(d-6)(-6 s_{23}^2 d + 24 s_{13}^2 + 2 s_{13} s_{23} d^2 - 40 s_{13} s_{23} d - 14 s_{13}^2 d + s_{13}^2 d^2 + \\
&\qquad\qquad + 8 s_{23}^2 + s_{23}^2 d^2 + 192 s_{13} s_{23})\mathcal{D}_2^\dagger - s_{13} s_{23}(-s_{13} + s_{23})\mathcal{D}_5^\dagger + \\
&\quad + (-2 s_{13}^2 + s_{13}^2 d + s_{23}^2 d - 24 s_{13} s_{23} - 2 s_{23}^2 + 2 s_{13} s_{23} d)\mathcal{D}_6^\dagger - \\
&\quad - 6(16 s_{13}^2 + 144 s_{13} s_{23} - 36 s_{13} s_{23} d + 2 s_{13} s_{23} d^2 - 10 s_{13}^2 d \\
&\qquad - 6 s_{23}^2 d + 8 s_{23}^2 + s_{23}^2 d^2 + s_{13}^2 d^2)\mathcal{D}_4^\dagger\Big), \qquad (2.15)
\end{aligned}
$$

where $\mathcal{D}_i^\dagger$ is the complex conjugate of the Dirac structures defined in eq. (2.11).

2.3 Reconstruction of the matrix elements

Starting from the tensor coefficients, the interference (or square) of the amplitude of eq. (2.12) can be easily extracted and we find that,

$$
\begin{aligned}
\sum_{\text{spins}} \langle M|M\rangle =& 2 s_{13}^2 \left|B_1(s_{12}, s_{23}) + \delta_{qQ} B_1(s_{23}, s_{12})\right|^2 + \\
& + 2 s_{23}^2 \left|B_2(s_{12}, s_{23})\right|^2 + 2\delta_{qQ} s_{12}^2 \left|B_2(s_{23}, s_{12})\right|^2 + \mathcal{O}(\epsilon) \qquad (2.16)
\end{aligned}
$$

where the combinations of tensor coefficients B_1 and B_2 are

$$
\begin{aligned}
B_1(s_{12}, s_{23}) =& 2A_1(s_{12}, s_{23}) - s_{23} A_2(s_{12}, s_{23}) + 32 A_3(s_{12}, s_{23}) + 4 s_{13} A_4(s_{12}, s_{23}) + \\
& + 512 A_5(s_{12}, s_{23}) + 64 s_{13} A_6(s_{12}, s_{23}), \\
B_2(s_{12}, s_{23}) =& 2A_1(s_{12}, s_{23}) + s_{13} A_2(s_{12}, s_{23}) + 8 A_3(s_{12}, s_{23}) + \\
& + 32 A_5(s_{12}, s_{23}) + 16 s_{13} A_6(s_{12}, s_{23}). \qquad (2.17)
\end{aligned}
$$

As mentioned earlier, $\delta_{qQ} = 1$ for identical quarks and is zero otherwise. Note that the tensor coefficients are in general matrices in colour space and the colour sum has still to be performed in evaluating, for example, $A_1 A_1^\dagger$. The functions $B_i(s_{23}, s_{12})$ are obtained from $B_i(s_{12}, s_{23})$ by exchanging the colour indices of the antiquarks as well as p_2 and p_4. Additional terms of order ϵ and ϵ^2 due to the d-dimensional nature of the Lorentz indices present in the six Dirac structures are straightforwardly generated. The resulting formula is, however, rather lengthy and we do not give it here.

Note that the squared amplitude is conventionally denoted by [47, 1, 2]

$$
\sum_{\text{spins}} \langle M|M\rangle = \mathcal{A}(s_{12}, s_{23}) + \delta_{qQ}\mathcal{A}(s_{23}, s_{12}) + \delta_{qQ}\mathcal{B}(s_{12}, s_{23}), \qquad (2.18)
$$

and by inspection we see that

$$
\begin{aligned}
\mathcal{A}(s_{12}, s_{23}) =& 2 s_{13}^2 \left|B_1(s_{12}, s_{23})\right|^2 + 2 s_{23}^2 \left|B_2(s_{12}, s_{23})\right|^2 + \mathcal{O}(\epsilon), \\
\mathcal{B}(s_{12}, s_{23}) =& 2 s_{13}^2 \Big(B_1(s_{12}, s_{23}) B_1^\dagger(s_{23}, s_{12}) + B_1(s_{23}, s_{12}) B_1^\dagger(s_{12}, s_{23})\Big) + \mathcal{O}(\epsilon). \qquad (2.19)
\end{aligned}
$$

3. The perturbative expansion of the tensor coefficients

The amplitude $|\mathcal{M}\rangle$ defined in eq. (2.10) has the perturbative expansion of the form,

$$|\mathcal{M}\rangle = 4\pi\,\alpha_s \left[|\mathcal{M}^{(0)}\rangle + \left(\frac{\alpha_s}{2\pi}\right) |\mathcal{M}^{(1)}\rangle + \left(\frac{\alpha_s}{2\pi}\right)^2 |\mathcal{M}^{(2)}\rangle + \mathcal{O}(\alpha_s^3) \right], \tag{3.1}$$

where $|\mathcal{M}^{(n)}\rangle$ denotes the n-loop contribution to the amplitude. Similarly, the coefficients have perturbative expansions of the form

$$A_I(s_{12}, s_{23}) = 4\pi\,\alpha_s \left[A_I^{(0)}(s_{12}, s_{23}) + \left(\frac{\alpha_s}{2\pi}\right) A_I^{(1)}(s_{12}, s_{23}) + \left(\frac{\alpha_s}{2\pi}\right)^2 A_I^{(2)}(s_{12}, s_{23}) + \mathcal{O}(\alpha_s^3) \right]. \tag{3.2}$$

The n-loop perturbative coefficients are vectors in colour space and can be further decomposed as,

$$A_I^{(n)}(s_{12}, s_{23}) = \sum_{i=1}^{2} \mathcal{C}_i\, A_I^{(n),[i]}(s_{12}, s_{23}), \tag{3.3}$$

where

$$\mathcal{C}_1 = \boldsymbol{\delta}_{i_3 i_2} \boldsymbol{\delta}_{i_1 i_4}, \qquad \mathcal{C}_2 = \boldsymbol{\delta}_{i_3 i_4} \boldsymbol{\delta}_{i_1 i_2}, \tag{3.4}$$

and indices $i_1, \ldots, i_4$ denote colour labels in the fundamental representation. In evaluating the squared amplitudes, we will encounter the colour sum $\sum_{\text{colours}} \mathcal{C}_i \mathcal{C}_j^\dagger$ for $1 \le i,j \le 2$ which is given by the matrix $\mathcal{CC}_{ij}$ where

$$\mathcal{CC}_{ij} = \begin{pmatrix} N^2 & N \\ N & N^2 \end{pmatrix}. \tag{3.5}$$

As in previous work, we use QGRAF [48] to generate the Feynman diagrams and then use MAPLE [49] and FORM [50] to manipulate the Dirac structures and perform the d-dimensional traces. When the projectors are applied to the individual Feynman diagrams, and the traces taken, we are left with a collection of two-loop scalar integrals with different topologies and powers of propagators. It is now well known (see for example [51, 52, 53] and references therein), that these integrals can be reduced [54] to a handful of *master integrals* using integration-by-parts [55, 56] and Lorentz-invariance [57] identities. For the process at hand — two-loop box graphs with four on-shell legs — the relevant master integrals [58]–[67] have been calculated in the past few years using a wide variety of methods.

3.1 Tree level results

At tree level, only $A_1^{(0)}$ is non-zero,

$$\begin{aligned} A_1^{(0),[1]}(s_{12}, s_{23}) &= \frac{1}{2 s_{12}}, \\ A_1^{(0),[2]}(s_{12}, s_{23}) &= -\frac{1}{N}\, A_1^{(0),[1]}(s_{12}, s_{23}). \end{aligned} \tag{3.6}$$

Using the d-dimensional form of eq. (2.16) together with the colour sum of eq. (3.5) we find that the self-interference of the tree amplitude is given by the usual result

$$\sum_{\text{spins}} \langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle = 2(N^2 - 1) \left(\frac{s_{13}^2 + s_{23}^2 - \epsilon s_{12}^2}{s_{12}^2} \right). \tag{3.7}$$

Similarly the interference of the identical quark amplitudes is given by

$$\sum_{\text{spins}} -\langle\overline{\mathcal{M}}^{(0)}|\mathcal{M}^{(0)}\rangle - \langle\mathcal{M}^{(0)}|\overline{\mathcal{M}}^{(0)}\rangle = -4\left(\frac{N^2-1}{N}\right)(1-\epsilon)\ \left(\frac{s_{13}^2}{s_{12}s_{23}}+\epsilon\right). \tag{3.8}$$

3.2 One-loop results

At one-loop, the first four Dirac structures $\mathcal{D}_1 \ldots \mathcal{D}_4$ contribute and the unrenormalised coefficients can be expressed in terms of the one-loop box integral in $d = 6-2\epsilon$ dimensions, $\mathrm{Box}^6(s_{ij}, s_{ik})$, and the one-loop bubble, $\mathrm{Bub}(s_{ij})$. To all orders in $\epsilon = (4-d)/2$, we find that,

$$\begin{aligned}
A_1^{(1),[1]}(s_{12},s_{23}) = &+\frac{(-2s_{23}+s_{12})\,[\mathrm{Bub}(s_{13})-\,\mathrm{Bub}(s_{12})]}{2s_{23}s_{12}N}+\frac{[\mathrm{Bub}(s_{13})-\,\mathrm{Bub}(s_{12})]}{Ns_{12}\epsilon}+\\
&+\frac{(N^2-2)(-2s_{23}+s_{12})\,[\mathrm{Bub}(s_{23})-\,\mathrm{Bub}(s_{12})]}{4s_{23}s_{12}N}+\\
&+\frac{(N^2-2)\,[\mathrm{Bub}(s_{23})-\,\mathrm{Bub}(s_{12})]}{2Ns_{12}\epsilon}-\\
&-\frac{(-18\epsilon N^2+9\epsilon^2N^2+6N^2+7\epsilon-8\epsilon^2+4\epsilon^3-6)\,\mathrm{Bub}(s_{12})}{4Ns_{12}(-3+2\epsilon)\epsilon}+\\
&+\frac{(\epsilon-1)N_F\,\mathrm{Bub}(s_{12})}{2s_{12}(-3+2\epsilon)}-\\
&-\frac{(N^2-2)(5s_{12}s_{23}+2s_{23}^2+s_{12}^2)\epsilon\,\mathrm{Box}^6(s_{12},s_{23})}{4s_{23}s_{12}N}-\\
&-\frac{(-2s_{23}^2+s_{12}^2)\epsilon\,\mathrm{Box}^6(s_{12},s_{13})}{2s_{23}s_{12}N}-\frac{s_{13}(-2s_{23}+s_{12})\,\mathrm{Box}^6(s_{12},s_{13})}{2s_{23}s_{12}N}+\\
&+\frac{(N^2-2)(2s_{12}+s_{23})\,\mathrm{Box}^6(s_{12},s_{23})}{2s_{12}N}\\
A_2^{(1),[1]}(s_{12},s_{23}) = &+\frac{2\,[\mathrm{Bub}(s_{13})-\,\mathrm{Bub}(s_{12})]}{s_{23}s_{12}N}-\\
&-\frac{(N^2-2)(2s_{23}+s_{12})\,[\mathrm{Bub}(s_{23})-\,\mathrm{Bub}(s_{12})]}{2Ns_{12}s_{13}s_{23}}-\\
&-\frac{(N^2-2)\epsilon\,[\mathrm{Bub}(s_{23})-\,\mathrm{Bub}(s_{12})]}{2Ns_{13}s_{23}}+\\
&+\frac{s_{12}(N^2-2)\epsilon^2\,\mathrm{Box}^6(s_{12},s_{23})}{2Ns_{13}s_{23}}+\\
&+\frac{(N^2-2)(2s_{23}+s_{12})(s_{12}-s_{23})\epsilon\,\mathrm{Box}^6(s_{12},s_{23})}{2Ns_{12}s_{13}s_{23}}+\\
&+\frac{2s_{13}\epsilon\,\mathrm{Box}^6(s_{12},s_{13})}{s_{23}s_{12}N}+\\
&+\frac{2\,\mathrm{Box}^6(s_{12},s_{13})}{Ns_{23}}-\frac{(N^2-2)\,\mathrm{Box}^6(s_{12},s_{23})}{2Ns_{13}}\\
A_3^{(1),[1]}(s_{12},s_{23}) = &+\frac{\mathrm{Box}^6(s_{12},s_{13})}{8N}-\frac{(N^2-2)\,\mathrm{Box}^6(s_{12},s_{23})}{16N}
\end{aligned}$$

JHEP04(2004)021

$$
\begin{aligned}
A_4^{(1),[1]}(s_{12},s_{23}) =& -\frac{[\mathrm{Bub}(s_{13})-\mathrm{Bub}(s_{12})]}{4Ns_{13}s_{23}} - \frac{(N^2-2)\,[\mathrm{Bub}(s_{23})-\mathrm{Bub}(s_{12})]}{8Ns_{13}s_{23}} + \\
&+\frac{\mathrm{Box}^6(s_{12},s_{13})}{4Ns_{23}} + \frac{(N^2-2)\,\mathrm{Box}^6(s_{12},s_{23})}{8Ns_{13}} + \\
&+\frac{s_{12}(N^2-2)\epsilon\,\mathrm{Box}^6(s_{12},s_{23})}{8Ns_{13}s_{23}} + \frac{s_{12}\epsilon\,\mathrm{Box}^6(s_{12},s_{13})}{4Ns_{13}s_{23}} \\
A_1^{(1),[2]}(s_{12},s_{23}) =& -\frac{(N^2+1)(-2s_{23}+s_{12})\,[\mathrm{Bub}(s_{13})-\mathrm{Bub}(s_{12})]}{4N^2s_{12}s_{23}} - \\
&-\frac{(N^2+1)\,[\mathrm{Bub}(s_{13})-\mathrm{Bub}(s_{12})]}{2N^2\epsilon s_{12}} + \\
&+\frac{(-2s_{23}+s_{12})\,[\mathrm{Bub}(s_{23})-\mathrm{Bub}(s_{12})]}{4N^2s_{12}s_{23}} + \frac{[\mathrm{Bub}(s_{23})-\mathrm{Bub}(s_{12})]}{2N^2\epsilon s_{12}} + \\
&+\frac{(-18\epsilon N^2+9\epsilon^2N^2+6N^2+7\epsilon-8\epsilon^2+4\epsilon^3-6)\,\mathrm{Bub}(s_{12})}{4s_{12}(-3+2\epsilon)N^2\epsilon} - \\
&-\frac{(\epsilon-1)N_F\,\mathrm{Bub}(s_{12})}{2Ns_{12}(-3+2\epsilon)} - \frac{(5s_{12}s_{23}+2s_{23}^2+s_{12}^2)\epsilon\,\mathrm{Box}^6(s_{12},s_{23})}{4N^2s_{12}s_{23}} + \\
&+\frac{(N^2+1)(-2s_{23}^2+s_{12}^2)\epsilon\,\mathrm{Box}^6(s_{12},s_{13})}{4N^2s_{12}s_{23}} + \\
&+\frac{s_{13}(N^2+1)(-2s_{23}+s_{12})\,\mathrm{Box}^6(s_{12},s_{13})}{4N^2s_{12}s_{23}} + \\
&+\frac{(2s_{12}+s_{23})\,\mathrm{Box}^6(s_{12},s_{23})}{2N^2s_{12}} \\
A_2^{(1),[2]}(s_{12},s_{23}) =& -\frac{(N^2+1)\,[\mathrm{Bub}(s_{13})-\mathrm{Bub}(s_{12})]}{N^2s_{12}s_{23}} - \\
&-\frac{(2s_{23}+s_{12})\,[\mathrm{Bub}(s_{23})-\mathrm{Bub}(s_{12})]}{2N^2s_{12}s_{13}s_{23}} - \\
&-\frac{\epsilon\,[\mathrm{Bub}(s_{23})-\mathrm{Bub}(s_{12})]}{2N^2s_{13}s_{23}} + \frac{s_{12}\epsilon^2\,\mathrm{Box}^6(s_{12},s_{23})}{2N^2s_{13}s_{23}} - \\
&-\frac{(N^2+1)\,\mathrm{Box}^6(s_{12},s_{13})}{N^2s_{23}} - \frac{\mathrm{Box}^6(s_{12},s_{23})}{2N^2s_{13}} + \\
&+\frac{(2s_{23}+s_{12})(s_{12}-s_{23})\epsilon\,\mathrm{Box}^6(s_{12},s_{23})}{2N^2s_{12}s_{13}s_{23}} - \frac{(N^2+1)s_{13}\epsilon\,\mathrm{Box}^6(s_{12},s_{13})}{N^2s_{12}s_{23}} \\
A_3^{(1),[2]}(s_{12},s_{23}) =& -\frac{(N^2+1)\,\mathrm{Box}^6(s_{12},s_{13})}{16N^2} - \frac{\mathrm{Box}^6(s_{12},s_{23})}{16N^2} \\
A_4^{(1),[2]}(s_{12},s_{23}) =& +\frac{(N^2+1)\,[\mathrm{Bub}(s_{13})-\mathrm{Bub}(s_{12})]}{8N^2s_{13}s_{23}} - \frac{[\mathrm{Bub}(s_{23})-\mathrm{Bub}(s_{12})]}{8N^2s_{13}s_{23}} - \\
&-\frac{(N^2+1)\,\mathrm{Box}^6(s_{12},s_{13})}{8N^2s_{23}} + \frac{\mathrm{Box}^6(s_{12},s_{23})}{8N^2s_{13}} + \\
&+\frac{s_{12}\epsilon\,\mathrm{Box}^6(s_{12},s_{23})}{8N^2s_{13}s_{23}} - \frac{s_{12}(N^2+1)\epsilon\,\mathrm{Box}^6(s_{12},s_{13})}{8N^2s_{13}s_{23}}\,. \qquad (3.9)
\end{aligned}
$$

Explicit expansions in ϵ for the $d=4-2\epsilon$ one-loop bubble and $d=6-2\epsilon$ one-loop box graphs through to $\mathcal{O}(\epsilon^2)$ can be found, for example, in the appendix of ref. [30].

3.3 Ultraviolet renormalisation

The renormalisation of the matrix element in the $\overline{\text{MS}}$ scheme is carried out by replacing the bare coupling α_0 with the renormalised coupling $\alpha_s \equiv \alpha_s(\mu^2)$, evaluated at the renormalisation scale μ^2 according to eq. (2.6).

We denote the i-loop contribution to the unrenormalised tensor coefficients by $A_I^{(i),\text{un}}$, using the same normalization as for the decomposition of the tensor coefficients in eq. (3.1). The renormalised coefficients are then obtained as

$$\begin{aligned}
A_I^{(0)}(s_{12},s_{23}) &= A_I^{(0),\text{un}}(s_{12},s_{23}), \\
A_I^{(1)}(s_{12},s_{23}) &= S_\epsilon^{-1} A_I^{(1),\text{un}}(s_{12},s_{23}) - \frac{\beta_0}{\epsilon} A_I^{(0),\text{un}}(s_{12},s_{23})\,, \\
A_I^{(2)}(s_{12},s_{23}) &= S_\epsilon^{-2} A_I^{(2),\text{un}}(s_{12},s_{23}) - \frac{2\beta_0}{\epsilon} S_\epsilon^{-1} A_I^{(1),\text{un}}(s_{12},s_{23}) - \\
&\quad - \frac{1}{2}\left(\frac{\beta_1}{\epsilon} - \frac{2\beta_0^2}{\epsilon^2}\right) A_I^{(0),\text{un}}(s_{12},s_{23})\,.
\end{aligned} \tag{3.10}$$

3.4 Infrared behaviour of the tensor coefficients

After performing ultraviolet renormalisation, the amplitudes still contain singularities which are of infrared origin and will be analytically cancelled by those occurring in radiative processes of the same order. It has become common practice to organize the infrared pole structure of the one- and two-loop contributions renormalised in the $\overline{\text{MS}}$ scheme in terms of the tree and renormalised one-loop amplitudes multiplied by singular operators according to Catani [42]. Precisely the same procedure applies to the tensor coefficients.

In particular, the infrared behaviour of the one-loop coefficients is given by

$$A_I^{(1)}(s_{12},s_{23}) = \boldsymbol{I}^{(1)}(\epsilon,s_{12},s_{23}) A_I^{(0)}(s_{12},s_{23}) + A_I^{(1),\text{finite}}(s_{12},s_{23})\,, \tag{3.11}$$

while the two-loop singularity structure is

$$\begin{aligned}
A_I^{(2)}(s_{12},s_{23}) &= \boldsymbol{I}^{(2)}(\epsilon,s_{12},s_{23}) A_I^{(0)}(s_{12},s_{23}) + \boldsymbol{I}^{(1)}(\epsilon,s_{12},s_{23}) A_I^{(1)}(s_{12},s_{23}) + \\
&\quad + A_I^{(2),\text{finite}}(s_{12},s_{23})\,,
\end{aligned} \tag{3.12}$$

where $\boldsymbol{I}^{(2)}(\epsilon,s_{12},s_{23})$ is given by,

$$\begin{aligned}
\boldsymbol{I}^{(2)}(\epsilon,s_{12},s_{23}) &= -\frac{1}{2}\boldsymbol{I}^{(1)}(\epsilon,s_{12},s_{23})\boldsymbol{I}^{(1)}(\epsilon,s_{12},s_{23}) - \frac{\beta_0}{\epsilon}\boldsymbol{I}^{(1)}(\epsilon,s_{12},s_{23}) + \\
&\quad + e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon} + K\right)\boldsymbol{I}^{(1)}(2\epsilon,s_{12},s_{23}) + \boldsymbol{H}^{(2)}(\epsilon,s_{12},s_{23})
\end{aligned} \tag{3.13}$$

and the constant K is

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{10}{9} T_R N_F\,. \tag{3.14}$$

The implicit dependence of $\boldsymbol{I}^{(1)}$, $\boldsymbol{I}^{(2)}$ and $\boldsymbol{H}^{(2)}$ on $s_{13} = -s_{12} - s_{23}$ has been suppressed.

In QCD the singular $\boldsymbol{I}^{(1)}(\epsilon)$ operator is a matrix in colour space and is given by [42]

$$\boldsymbol{I}^{(1)}(\epsilon) = \frac{1}{2}\frac{e^{\epsilon\gamma}}{\Gamma(1-\epsilon)} \sum_i \nu_i^{sing}(\epsilon) \sum_{j\neq i} \mathbf{T}_i \cdot \mathbf{T}_j \left(-\frac{\mu^2}{2p_i\cdot p_j}\right)^{\epsilon} \tag{3.15}$$

where the sum runs over the pairs of external coloured particles involved in the process and the singular function is

$$\nu_i^{sing}(\epsilon) = \frac{1}{\epsilon^2} + \gamma_i \frac{1}{\epsilon}\,, \qquad \gamma_{q,\bar{q}} = \frac{3}{2}\,, \qquad \gamma_g = \frac{\beta_0}{C_A} \tag{3.16}$$

As usual, the colour charge algebra is defined as

$$\mathbf{T}_i \cdot \mathbf{T}_j = \begin{cases} \mathbf{T}_j \cdot \mathbf{T}_i & \text{if } i \neq j\,, \\ \mathbf{T}_i^2 = C_i & \text{otherwise}\,, \end{cases}$$

with the Casimir $C_i = C_F$ ($C_i = C_A$) if parton i is a quark (gluon) and where the colour charge $\mathbf{T}_i$ is t^a_{cb} ($-t^a_{cb}$) for a quark (anti-quark) and if_{cab} for a gluon.

For the colour basis for the four-quark process presented in eq. (3.3), we have the following operator

$$\boldsymbol{I}^{(1)}(\epsilon, s_{12}, s_{23}) = -\frac{e^{\epsilon\gamma}}{\Gamma(1-\epsilon)}\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right)\begin{pmatrix} \boldsymbol{A}(\epsilon, s_{12}, s_{23}) & \boldsymbol{B}(\epsilon, s_{12}, s_{23}) \\ \boldsymbol{B}(\epsilon, s_{23}, s_{12}) & \boldsymbol{A}(\epsilon, s_{23}, s_{12}) \end{pmatrix} \tag{3.17}$$

where

$$\boldsymbol{A}(\epsilon, s_{12}, s_{23}) = \frac{(N^2-1)}{N}\left(-\frac{\mu^2}{s_{23}}\right)^{\epsilon} + \frac{1}{N}\left[\left(-\frac{\mu^2}{s_{13}}\right)^{\epsilon} - \left(-\frac{\mu^2}{s_{12}}\right)^{\epsilon}\right], \tag{3.18}$$

$$\boldsymbol{B}(\epsilon, s_{12}, s_{23}) = \left[\left(-\frac{\mu^2}{s_{23}}\right)^{\epsilon} - \left(-\frac{\mu^2}{s_{13}}\right)^{\epsilon}\right]. \tag{3.19}$$

In eq. (3.13) the function $\boldsymbol{H}^{(2)}$ contains poles of $\mathcal{O}(1/\epsilon)$ and is process and renormalisation scheme dependent. The specific structure of this term was not given in ref. [42] for the general case, but in the case where a single $q\bar{q}$ pair is involved, $\boldsymbol{H}^{(2)}$ is related to the quark electromagnetic form factor. Subsequent work by Sterman and Tejeda-Yeomans [68] has shown that the form factor is in fact the backbone of the singularity structure of multi-loop QCD amplitudes.

The colour uncorrelated part of the $\boldsymbol{H}^{(2)}$ function has now been established for all $2 \to 2$ partonic processes by direct Feynman diagram evaluation of two-loop matrix elements [69, 1, 2, 30, 31, 70]. More precisely, it was found that each external coloured leg in the partonic process contributes independently to the matrix element given by

$$\langle\mathcal{M}_0|\boldsymbol{H}^{(2)}|\mathcal{M}_0\rangle = \frac{e^{\epsilon\gamma}}{4\epsilon\Gamma(1-\epsilon)} H^{(2)}\langle\mathcal{M}_0|\mathcal{M}_0\rangle\,. \tag{3.20}$$

For the processes we are interested in here,

$$H^{(2)} = 2H_q^{(2)} + 2H_{\bar{q}}^{(2)} \tag{3.21}$$

with

$$H_q^{(2)} = H_{\bar{q}}^{(2)} = \left(\frac{\pi^2}{2} - 6\zeta_3 - \frac{3}{8}\right)C_F^2 + \left(\frac{13}{2}\zeta_3 + \frac{245}{216} - \frac{23}{48}\pi^2\right)C_A C_F + \left(-\frac{25}{54} + \frac{\pi^2}{12}\right)T_R N_F C_F\,. \tag{3.22}$$

However, $\boldsymbol{H}^{(2)}$ also contains additional non-trivial colour correlations [42] which can be explored by calculating the hard scattering amplitude, rather than the interference of two-loop and tree amplitudes where the sum over colours causes the colour correlations to vanish. Such colour correlations are independent of the helicity configuration and have been identified in the two-loop helicity amplitudes for the four gluon and the two-quark two-gluon interactions which both involve four coloured particles [35, 36, 37]. According to the analysis of ref. [68] the colour-correlated contributions to $\hat{\mathbf{H}}^{(2)}$ are due to soft gluons[2] and are directly related to soft anomalous dimension matrices obtained by studying the evolution of colour exchange in QCD hard scattering (see [72] and references therein). They are therefore also independent of the type of (coloured) external particle.

For the four-quark processes studied here, we find that the colour correlations are indeed helicity independent and are given by

$$\boldsymbol{H}^{(2)}(\epsilon, s_{12}, s_{23}) = \frac{e^{\epsilon\gamma}}{4\epsilon\Gamma(1-\epsilon)}\left[\left(-\frac{\mu^2}{s_{12}}\right)^{2\epsilon} + \left(-\frac{\mu^2}{s_{23}}\right)^{2\epsilon} - \left(-\frac{\mu^2}{s_{13}}\right)^{2\epsilon}\right] \times \\ \times \left(H^{(2)}\ \mathbf{1} + \widehat{\mathbf{H}}^{(2)}(s_{12}, s_{23})\right) \tag{3.23}$$

with $H^{(2)}$ as given in eq. (3.21) and

$$\widehat{\mathbf{H}}^{(2)}(s_{12}, s_{23}) = -4\ i\ f_{abc}\ \boldsymbol{T}^{a[1]}\boldsymbol{T}^{b[2]}\boldsymbol{T}^{c[3]} \ln\left(\frac{-s_{12}}{-s_{23}}\right) \ln\left(\frac{-s_{23}}{-s_{13}}\right) \ln\left(\frac{-s_{13}}{-s_{12}}\right). \tag{3.24}$$

The structure of $\widehat{\mathbf{H}}^{(2)}(s_{12}, s_{23})$ is in complete agreement with the explicit results found in ref. [35, 36, 37] and confirms that the colour correlations are independent of the external particles. Note that the square bracket in eq. (3.23) is, to some extent, arbitrary and other choices may be equally valid. We have made this particular choice partly to match the choice made in refs. [1, 2], but also to ensure that no additional finite contribution from the colour correlation term is generated when the "squared" matrix elements for identical quark scattering are reconstructed.[3]

Note that the matrix $f_{abc}\,\boldsymbol{T}^{a[1]}\boldsymbol{T}^{b[2]}\boldsymbol{T}^{c[3]}$ collects all colour weights that result from the analysis of triple gluon vertices connecting any three different external partons, projected into the colour basis given in eq. (3.3), so that for the four-quark process,

$$i\, f_{abc}\,\boldsymbol{T}^{a[1]}\boldsymbol{T}^{b[2]}\boldsymbol{T}^{c[3]} = -\frac{1}{4}\begin{pmatrix} -1 & -N \\ N & 1 \end{pmatrix}. \tag{3.25}$$

4. Helicity amplitudes

The helicity amplitudes $|\mathcal{M}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4}$ can be obtained from the general d-dimensional tensor of eq. (2.10) by setting the dimensionality of the Lorentz matrices to be four and using

[2] Note that these type of colour correlations between three different partons also appeared in the analysis of higher-order contributions to the soft-gluon current in ref. [71].

[3] Colour automatically kills the contribution from $\widehat{\mathbf{H}}^{(2)}$ when the quarks are distinct.

standard four-dimensional helicity techniques. This corresponds to working in the 't Hooft-Veltman scheme. We use the standard convention of denoting the two helicity states of a four-dimensional light-like spinor $\psi(p)$ by,

$$\psi_{\pm}(p) = \frac{1}{2}(1 \pm \gamma_5)\psi(p)\,, \tag{4.1}$$

with the further notation,

$$|p\pm\rangle = \psi_{\pm}(p)\,, \qquad \langle p\pm| = \overline{\psi_{\pm}(p)}\,. \tag{4.2}$$

Particles may thus be crossed to the initial state by reversing the sign of the helicity. The basic quantity is the spinor product,

$$\langle pq\rangle = \langle p - |q+\rangle\,, \qquad [pq] = [p + |q-]\,, \tag{4.3}$$

such that

$$\langle pq\rangle[qp] = 2p.q\,. \tag{4.4}$$

Fixing the helicity of the (final-state) quark q to be positive, $\lambda_1 = +$ (and therefore the helicity of the antiquark $\bar{q}$ is negative, $\lambda_2 = -$) we find that, up to a helicity dependent phase, the two independent helicity amplitudes are given by

$$\begin{aligned} |\mathcal{M}\rangle_{+-+-} &\propto s_{13}\left(B_1(s_{12},s_{23}) + \delta_{qQ}B_1(s_{23},s_{12})\right)\,,\\ |\mathcal{M}\rangle_{+--+} &\propto s_{23}B_2(s_{12},s_{23})\,, \end{aligned} \tag{4.5}$$

where the combinations of tensor coefficients B_1 and B_2 are defined in eq. (2.17). In addition, when the quarks are identical, there is the additional helicity amplitude,

$$|\mathcal{M}\rangle_{++--} \propto \delta_{qQ}\; s_{12}B_2(s_{23},s_{12})\,. \tag{4.6}$$

Helicity amplitudes where the quark helicity is negative are obtained through parity,

$$|\mathcal{M}\rangle_{--\lambda_2-\lambda_3-\lambda_4} = \left(|\mathcal{M}\rangle_{+\lambda_2\lambda_3\lambda_4}\right)^*\,. \tag{4.7}$$

The helicity amplitudes are functions of the external scales, however the dependence on s_{12} and s_{23} (together with $s_{13} = -s_{12} - s_{23}$) has been suppressed.

By inspection, we see that "squaring" and summing over the helicities generates the interference of amplitudes given in eqs. (3.7) and (3.8). Thus, the helicity amplitudes can be obtained directly from a Feynman diagram calculation by projecting out the individual tensor coefficients and taking the appropriate linear combination.

Just as for the individual tensor coefficients, the helicity amplitudes can be perturbatively decomposed as,

$$\begin{aligned} |\mathcal{M}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} = 4\pi\,\alpha_s\Bigg[&|\mathcal{M}^{(0)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} + \left(\frac{\alpha_s}{2\pi}\right)|\mathcal{M}^{(1)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} +\\ &+ \left(\frac{\alpha_s}{2\pi}\right)^2|\mathcal{M}^{(2)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} + \mathcal{O}(\alpha_s^3)\Bigg]\,. \end{aligned} \tag{4.8}$$

As in eq. (3.10) the renormalised helicity amplitudes are obtained from the unrenormalised amplitudes by

$$\begin{aligned}
|\mathcal{M}^{(0)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} &= |\mathcal{M}^{(0),\mathrm{un}}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4}, \\
|\mathcal{M}^{(1)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} &= S_\epsilon^{-1}|\mathcal{M}^{(1),\mathrm{un}}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} - \frac{\beta_0}{\epsilon}|\mathcal{M}^{(0),\mathrm{un}}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4}, \\
|\mathcal{M}^{(2)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} &= S_\epsilon^{-2}|\mathcal{M}^{(2),\mathrm{un}}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} - \frac{2\beta_0}{\epsilon}S_\epsilon^{-1}|\mathcal{M}^{(1),\mathrm{un}}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} - \\
&\quad -\frac{1}{2}\left(\frac{\beta_1}{\epsilon} - \frac{2\beta_0^2}{\epsilon^2}\right)|\mathcal{M}^{(0),\mathrm{un}}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4},
\end{aligned} \tag{4.9}$$

while the infrared singularity structure is given by,

$$|\mathcal{M}^{(1)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} = \boldsymbol{I}^{(1)}(\epsilon, s_{12}, s_{23})|\mathcal{M}^{(0)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} + |\mathcal{M}^{(1),\mathrm{finite}}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4}, \tag{4.10}$$

$$\begin{aligned}
|\mathcal{M}^{(2)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} &= \boldsymbol{I}^{(2)}(\epsilon, s_{12}, s_{23})|\mathcal{M}^{(0)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} + \boldsymbol{I}^{(1)}(\epsilon, s_{12}, s_{23})|\mathcal{M}^{(1)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} + \\
&\quad + |\mathcal{M}^{(2),\mathrm{finite}}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4},
\end{aligned} \tag{4.11}$$

where the operators $\boldsymbol{I}^{(i)}$ are the process dependent matrices in colour space given in section 3.4.

Similarly, the n-loop helicity amplitudes are vectors in colour space and can be further decomposed as in eq. (3.3),

$$|\mathcal{M}^{(n)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} = \sum_{i=1}^{2} \mathcal{C}_i\,|\mathcal{M}^{(n),[i]}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4}. \tag{4.12}$$

4.1 Helicity amplitudes for physical processes

The physically relevant scattering amplitudes we wish to describe are the unlike quark processes,

$$s: \quad q(p_2,+) + \bar{q}(p_1,-) \to Q(p_3,\lambda_3) + \bar{Q}(p_4,\lambda_4), \tag{4.13}$$

$$t: \quad q(p_2,+) + \bar{Q}(p_1,\lambda_1) \to q(p_3,+) + \bar{Q}(p_4,\lambda_4), \tag{4.14}$$

$$t': \quad q(p_2,+) + Q(p_1,\lambda_1) \to q(p_3,+) + Q(p_4,\lambda_4), \tag{4.15}$$

$$u: \quad q(p_2,+) + Q(p_1,\lambda_1) \to Q(p_3,\lambda_3) + q(p_4,+). \tag{4.16}$$

For convenience, we denote each of the processes of eqs. (4.13)–(4.16) as belonging to a particular channel (e.g. eq. (4.13) is the s-channel and so on) where s, t and u are the usual Mandelstam variables, $s = s_{12} > 0$, $t = s_{23} < 0$, $u = s_{13} < 0$ and $s+t+u = 0$. Processes where the initial state quark has negative helicity are obtained by a parity transformation, while processes with $\bar{q}(p_2)$ in the initial state are obtained by charge conjugation. Note that the t'-channel amplitudes can be obtained by exchanging $1 \leftrightarrow 4$ in the t-channel amplitudes or by exchanging $3 \leftrightarrow 4$ in the u-channel amplitudes.

To present the helicity amplitudes for the various processes, it is convenient to organise the amplitude where particles 1 and 2 are in the initial state in terms of a spinor factor $\mathcal{S}$

JHEP04(2004)021

and the colour factor $\mathcal{C}$ so that,

$$|\mathcal{M}_c^{(n)}\rangle_{\lambda_2\lambda_1\lambda_3\lambda_4} = \sum_i \mathcal{S}^{[i]}_{c\lambda_2\lambda_1\lambda_3\lambda_4} \times \mathcal{C}_{ci} \times |\widehat{\mathcal{M}}_c^{(n),[i]}\rangle_{\lambda_2\lambda_1\lambda_3\lambda_4}\,, \tag{4.17}$$

where c $(=s,\ t,\ u)$ denotes the channel and the sum runs over the two colour structures. Explicitly, we have

$$\mathcal{C}_{s1} = \boldsymbol{\delta}_{i_3 i_2}\boldsymbol{\delta}_{i_1 i_4}\,, \qquad \mathcal{C}_{s2} = \boldsymbol{\delta}_{i_3 i_4}\boldsymbol{\delta}_{i_1 i_2}\,. \tag{4.18}$$

The u- and t-channel colour factors are obtained by crossing symmetry,

$$\mathcal{C}_{ti} = \mathcal{C}_{si} \quad (1 \leftrightarrow 3)\,, \qquad \mathcal{C}_{ui} = \mathcal{C}_{si} \quad (1 \leftrightarrow 4)\,, \tag{4.19}$$

while the t'-channel colour factor is given by,

$$\begin{aligned} \mathcal{C}_{t'i} &= \mathcal{C}_{ti} \quad (1 \leftrightarrow 4) \\ &= \mathcal{C}_{ui} \quad (3 \leftrightarrow 4)\,. \end{aligned} \tag{4.20}$$

We note that $\mathcal{C}_{t1} = \mathcal{C}_{s2}$, $\mathcal{C}_{t2} = \mathcal{C}_{s1}$ and $\mathcal{C}_{t'1} = \mathcal{C}_{u2}$, $\mathcal{C}_{t'2} = \mathcal{C}_{u1}$.

There is considerable freedom in the definition of the helicity dependent spinor factors. For the s-channel, for process (4.13) we choose,

$$\begin{aligned} \mathcal{S}^{[i]}_{s+-+-} &= -i\frac{[13]\langle 42\rangle}{s_{12}}\,, \\ \mathcal{S}^{[i]}_{s+--+} &= -i\frac{[14]\langle 32\rangle}{s_{12}}\,. \end{aligned} \tag{4.21}$$

The spinor prefactors for the other channels are also obtained by crossing symmetry,

$$\mathcal{S}^{[i]}_{t\lambda_2\lambda_1\lambda 3\lambda_4} = \mathcal{S}^{[i]}_{s\lambda_2-\lambda_3-\lambda_1\lambda_4} \quad (p_1 \leftrightarrow p_3)\,, \qquad \mathcal{S}^{[i]}_{u\lambda_2\lambda_1\lambda_3\lambda_4} = \mathcal{S}^{[i]}_{s\lambda_2-\lambda_4\lambda_3-\lambda_1} \quad (p_1 \leftrightarrow p_4)\,, \tag{4.22}$$

and,

$$\mathcal{S}^{[i]}_{t'\lambda_2\lambda_3\lambda_4\lambda_4} = \mathcal{S}^{[i]}_{u\lambda_2\lambda_1\lambda_4\lambda_3} \quad (p_3 \leftrightarrow p_4)\,. \tag{4.23}$$

The helicity amplitudes for the like-quark scattering processes,

$$st: \ \ q(p_2,+) + \bar{q}(p_1,-) \to q(p_3,\lambda_3) + \bar{q}(p_4,\lambda_4)\,, \tag{4.24}$$

$$ut': \ \ q(p_2,+) + q(p_1,\lambda_1) \to q(p_3,+) + q(p_4,\lambda_4)\,, \tag{4.25}$$

are derived directly from the unlike quark amplitudes. Here we have labelled the two processes according to the two channels that contribute. Explicitly, we have

$$|\mathcal{M}_{st}^{(n)}\rangle_{+-+-} = \ \ |\mathcal{M}_s^{(n)}\rangle_{+-+-} - |\mathcal{M}_t^{(n)}\rangle_{+-+-}, \tag{4.26}$$

$$|\mathcal{M}_{st}^{(n)}\rangle_{+--+} = \ \ |\mathcal{M}_s^{(n)}\rangle_{+-+-}\,, \tag{4.27}$$

$$|\mathcal{M}_{st}^{(n)}\rangle_{++++} = -|\mathcal{M}_t^{(n)}\rangle_{++++}\,, \tag{4.28}$$

$$|\mathcal{M}_{ut'}^{(n)}\rangle_{++++} = \ \ |\mathcal{M}_u^{(n)}\rangle_{++++} - |\mathcal{M}_{t'}^{(n)}\rangle_{++++}\,, \tag{4.29}$$

$$|\mathcal{M}_{ut'}^{(n)}\rangle_{+--+} = \ \ |\mathcal{M}_u^{(n)}\rangle_{+--+}\,, \tag{4.30}$$

$$|\mathcal{M}_{ut'}^{(n)}\rangle_{+-+-} = -|\mathcal{M}_{t'}^{(n)}\rangle_{+-+-}\,, \tag{4.31}$$

where the minus sign is due to the exchange of identical particles.

4.2 Tree level helicity amplitudes

At tree-level, inserting the results for the tensor coefficients given in section 3.1, we find the only non-vanishing helicity amplitudes in the s-channel are,

$$
\begin{aligned}
|\widehat{\mathcal{M}}_s^{(0),[1]}\rangle_{+-+-} &= |\widehat{\mathcal{M}}_s^{(0),[1]}\rangle_{+--+} = 1\,, \\
|\widehat{\mathcal{M}}_s^{(0),[2]}\rangle_{+-+-} &= |\widehat{\mathcal{M}}_s^{(0),[2]}\rangle_{+--+} = -\frac{1}{N}\,.
\end{aligned}
\tag{4.32}
$$

The appropriate spinor factors are obtained from eq. (4.21) while the colour factors are given by eq. (4.18). Tree helicity amplitudes for the other channels are obtained by the appropriate crossing symmetry.

4.3 One-loop helicity amplitudes

The one-loop helicity amplitudes are straightforwardly obtained by inserting the all-orders results for the tensor coefficients given in section 3.2 into eq. (4.5), expanding the one-loop bubble and box integrals around $\epsilon = 0$, and renormalising according to eq. (4.9). Explicit expansions in ϵ for the $d = 4 - 2\epsilon$ one-loop bubble and $d = 6 - 2\epsilon$ one-loop box graphs can be found, for example, in the appendix of ref. [30].

The finite remainders of the one-loop amplitudes defined through eq. (4.10) for the distinct quark scattering process can be decomposed according to the number of colours and massless quark flavours,

$$
\begin{aligned}
|\widehat{\mathcal{M}}_c^{(1),[1],\mathrm{finite}}\rangle &= N A_c^{(1),[1]} + \frac{1}{N} B_c^{(1),[1]} + N_F C_c^{(1),[1]} - \beta_0 S\, |\widehat{\mathcal{M}}_c^{(0),[1]}\rangle\,, \\
|\widehat{\mathcal{M}}_c^{(1),[2],\mathrm{finite}}\rangle &= A_c^{(1),[2]} + \frac{1}{N^2} B_c^{(1),[2]} + \frac{N_F}{N} C_c^{(1),[2]} - \beta_0 S\, |\widehat{\mathcal{M}}_c^{(0),[2]}\rangle\,,
\end{aligned}
\tag{4.33}
$$

where β_0 is defined in eq. (2.8). For clarity, the dependence on the helicities has been suppressed. Explicit expressions for $A, \ldots, C$ in the physical region, $s = s_{12} > 0$, $t = s_{23} < 0$ and $u = s_{13} < 0$, are given in appendix B. We have checked that the finite one-loop helicity amplitudes presented here are in agreement with those given in ref. [73].

4.4 Two-loop helicity amplitudes

The main results of this paper are the two-loop amplitudes for processes (4.13)–(4.16). As in the tree and one-loop cases, the helicity amplitudes can be directly extracted from the appropriate linear combination, eq. (4.5), of unrenormalised two-loop tensor coefficients obtained by direct evaluation of the projectors given in section 2.2 acting on two-loop Feynman diagrams. Renormalisation is achieved via eq. (4.9) and the two-loop master integrals expanded around $\epsilon = 0$. The finite remainder of the two-loop amplitudes are defined through eq. (4.11) and, for each process can be decomposed according to the number of colours and massless quark flavours,

$$
\begin{aligned}
|\widehat{\mathcal{M}}_c^{(2),[1],\mathrm{finite}}\rangle &= N^2 A_c^{(2),[1]} + B_c^{(2),[1]} + \frac{1}{N^2} C_c^{(2),[1]} + \\
&\quad + N N_F D_c^{(2),[1]} + \frac{N_F}{N} E_c^{(2),[1]} + N_F^2 F_c^{(2),[1]} -
\end{aligned}
$$

$$-2\beta_0 S\,|\widehat{\mathcal{M}}_c^{(1),[1],\text{finite}}\rangle - (\beta_1 S + \beta_0^2 S^2)\,|\widehat{\mathcal{M}}_c^{(0),[1]}\rangle\,, \tag{4.34}$$

$$\begin{aligned}|\widehat{\mathcal{M}}_c^{(2),[2],\text{finite}}\rangle &= N A_c^{(2),[2]} + \frac{1}{N}B_c^{(2),[2]} + \frac{1}{N^3}C_c^{(2),[2]} + \\ &\quad + N_F D_c^{(2),[2]} + \frac{N_F}{N^2}E_c^{(2),[2]} + \frac{N_F^2}{N}F_c^{(2),[2]} - \\ &\quad -2\beta_0 S\,|\widehat{\mathcal{M}}_c^{(1),[2],\text{finite}}\rangle - (\beta_1 S + \beta_0^2 S^2)\,|\widehat{\mathcal{M}}_c^{(0),[2]}\rangle\,.\end{aligned} \tag{4.35}$$

As in the previous section, the dependence on the helicities has been suppressed. Explicit expressions for $A, \ldots, E$ for the various processes in the physical region, $s = s_{12} > 0$, $t = s_{23} < 0$ and $u = s_{13} < 0$, are given in appendix A.

5. Comparison with previous results

In recovering the square, or interference, of amplitudes in a particular channel c, the contraction of the n-loop colour vector $|\mathcal{M}^{(n)}\rangle$ with a conjugate m-loop colour vector $\langle\mathcal{M}^{(m)}|$ obeys the rule

$$\langle\mathcal{M}^{(m)}|\mathcal{M}^{(n)}\rangle = \sum_{\text{helicities}}\sum_{\text{colours}} |\mathcal{M}_c^{(n)}\rangle^*_{\lambda_1\lambda_2\lambda_3\lambda_4}\,|\mathcal{M}_c^{(n)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4} \tag{5.1}$$

where $|\mathcal{M}_c^{(n)}\rangle_{\lambda_1\lambda_2\lambda_3\lambda_4}$ contains all of the spinor and colour information for a particular helicity. Expanding the colour and spinor factors $\mathcal{C}$ and $\mathcal{S}$, and dropping the explicit dependence on the helicities, we see that,

$$\begin{aligned}\langle\mathcal{M}^{(m)}|\mathcal{M}^{(n)}\rangle &= \sum_{\text{helicities}}\sum_{\text{colours}}\sum_{i,j} \mathcal{C}_{ci}^*\,\mathcal{C}_{cj}\,\mathcal{S}_c^{[i]*}\,\mathcal{S}_c^{[j]}\,|\widehat{\mathcal{M}}_c^{(n),[i]}\rangle^*\,|\widehat{\mathcal{M}}_c^{(n),[j]}\rangle \\ &= \sum_{\text{helicities}}\sum_{i,j} \mathcal{CC}_{ij}\,\mathcal{SS}_{ij}\,|\widehat{\mathcal{M}}_c^{(n),[i]}\rangle^*\,|\widehat{\mathcal{M}}_c^{(n),[j]}\rangle\,,\end{aligned} \tag{5.2}$$

where the colour sum matrix $\mathcal{CC}$ is defined in eq. (3.5).

In the s-channel, the helicity dependent spinor matrices $\mathcal{SS}_c = \mathcal{S}_c^{[i]*}\,\mathcal{S}_c^{[j]}$ are given by,

$$\mathcal{SS}_{s+-+-} = \frac{u^2}{s^2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \qquad \mathcal{SS}_{s+--+} = \frac{t^2}{s^2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}. \tag{5.3}$$

Similar matrices for the other helicities are obtained by a parity transformation, while corresponding matrices for the u and t-channels are obtained by crossing symmetry,

$$\mathcal{SS}_u = \mathcal{SS}_s(s \leftrightarrow t)\,, \qquad \mathcal{SS}_t = \mathcal{SS}_s(s \leftrightarrow u)\,, \tag{5.4}$$

and

$$\mathcal{SS}_{t'} = \mathcal{SS}_u(u \leftrightarrow t)\,. \tag{5.5}$$

Equations (5.1) and (5.2) can be used to recreate the full interference of tree and two-loop amplitudes. However, we notice that we can immediately separate the singularities from the finite parts using eq. (4.11). The finite remainder, denoted by $\mathcal{F}_{inite}^{2\times 0}(s,t,u)$ in refs. [1, 2] is thus given by,

$$2\,\mathrm{Re}\sum_{\text{helicities}}\sum_{i,j} \mathcal{CC}_{ij}\,\mathcal{SS}_{ij}\,|\widehat{\mathcal{M}}_c^{(0),[i]}\rangle^*\,|\widehat{\mathcal{M}}_c^{(2),[j],\text{finite}}\rangle\,. \tag{5.6}$$

Note that by using helicities, we are implicitly treating the external particle states in 4-dimensions — the HV scheme. However, the singular contributions to the renormalised amplitudes defined in eqs. (4.10), (4.11) are given relative to the tree amplitude and therefore any additional terms that would be produced in CDR are automatically removed. Treating the external states differently for both the n-loop and tree amplitudes does not alter the finite contribution. Direct application of eq. (5.1) for the real part of the tree and two-loop interference ($n = 0$ and $m = 2$) in the HV scheme therefore generates the same finite parts given in section 4.2 of ref. [1] for unlike quark-quark scattering and in section 3.2 of ref. [2] for like quark-quark scattering obtained using CDR.

Similarly, the square of one-loop graphs is obtained using $n = 1$ and $m = 1$ in eq. (5.1). In principle, the one-loop amplitude should be expanded through to $\mathcal{O}(\epsilon^2)$. However, as noted in [30, 35, 74], the finite self interference only contains logarithms (up to the fourth power), but does not contain triple and quartic polylogs. These terms naturally arise when the $\mathcal{O}(\epsilon)$ and $\mathcal{O}(\epsilon^2)$ terms in the expansion of the one-loop box are multiplied by singularities from the conjugate amplitude. Such terms appear in the Catani pole structure, $\langle\mathcal{M}^{(1)}|\boldsymbol{I}^{(1)}|\mathcal{M}^{(0)}\rangle$ where $|\mathcal{M}^{(1)}\rangle$ does need to be expanded through to $\mathcal{O}(\epsilon^2)$. The finite parts of the renormalised one-loop self interference remaining after Catani's prediction for the singularities has been removed should therefore be identical to those obtained by summing the finite parts using eq. (5.1). In this instance, the finite remainder denoted by $\mathcal{F}^{1\times 1}_{inite}(s,t,u)$ in [70] is obtained from

$$\sum_{\text{helicities}}\sum_{i,j}\mathcal{CC}_{ij}\,\mathcal{SS}_{ij}\,|\widehat{\mathcal{M}}_c^{(1),[i],\text{finite}}\rangle^*\,|\widehat{\mathcal{M}}_c^{(1),[j],\text{finite}}\rangle\,. \tag{5.7}$$

Unfortunately, the results of ref. [32] are not presented in quite the same way and a direct comparison is more difficult. However, the coefficients of the finite remainder for the one-loop squared amplitudes using the Catani pole structure are presented in ref. [74] and we have checked that one-loop square reconstructed from the helicity amplitudes (the HV scheme) correctly reproduces the coefficients given in appendix C of ref. [74].

6. Conclusions

In this paper we have presented the analytic expressions for the one- and two-loop helicity amplitudes for massless quark-quark scattering $q\bar{q} \to Q\bar{Q}$ and those processes related by crossing symmetry. These amplitudes were obtained by the construction of d-dimensional projection operators that extract the coefficients of the most general tensor representation for the amplitude, order by order in perturbation theory. Once the renormalised tensor coefficients are known, the renormalised helicity amplitudes can be constructed straightforwardly as a linear combination of the coefficients.

We applied the projection operators directly to the Feynman diagrams that contribute at each order in α_s. The projectors saturate the tensor structure of the Feynman diagram and yield a set of scalar integrals that can be related to a basis set of master integrals with the application of widely used reduction algorithms. In fact, we are using exactly the same tools as we did in [1, 2], except in that case, the projector was merely the conjugate tree amplitude. Because the projectors exist in d-dimensions, conventional dimensional regularisation is preserved and there is no ambiguity in dealing with γ_5. Once the general tensor coefficients are determined, the external particles can be treated in 4-dimensions (the 't Hooft-Veltman scheme) and standard helicity methods used to construct the helicity amplitudes.

By summing over helicities and colours, the full interference of tree and two-loop graphs can be reconstructed. In previous work [1, 2], the finite part has been separated from infrared singular parts using the Catani formalism [42]. In this procedure, the singular operators multiply tree and one-loop amplitudes. Changing the scheme for the external particles therefore changes the overall pole contribution. However, the finite remainder left after subtracting the poles is invariant under changing from the conventional dimensional regularisation scheme to the 't Hooft-Veltman scheme. Therefore, the finite parts of the helicity amplitudes presented here precisely reproduce the finite parts of the interference of tree and two-loop graphs given in [1, 2]. Similarly, the finite parts of the self interference of one-loop amplitudes can be reconstructed purely from the finite parts of the one-loop helicity amplitudes.

As in quark-gluon and gluon-gluon scattering, the presence of four coloured external particles gives rise to additional colour correlations proportional to $1/\epsilon$. While the existence of such additional non-trivial colour correlations was pointed out in ref. [42] and are expected on general grounds [68], the precise form of the colour structure of $\boldsymbol{H}^{(2)}$ was not predicted. So far, the results for gluon-gluon scattering [35] and quark-gluon scattering [36, 37] as well as those for quark-quark scattering presented in this paper support the form given in eqs. (3.23) and (3.24). These colour correlations vanish when the interference of tree and two-loop amplitudes is constructed.

As discussed in section 1, the more general goal is to produce a numerical NNLO program (similar to the NLO codes JETRAD [75, 76] and EKS [77, 78] that have been extensively used over the past decade) and to reduce the theoretical uncertainty on the jet cross section to a level that is competitive with the anticipated experimental measurements at the Tevatron and the LHC. Such a programme would naturally include the two-loop amplitudes presented here. At present there is intensive activity [79]–[84] towards achieving this goal and thereby testing QCD at the few percent level.

Acknowledgments

We thank Maria Elena Tejeda-Yeomans for help in comparing with the results of ref. [74] and Adrian Signer for helpful discussions. We gratefully acknowledge the financial support of PPARC via a Senior Fellowship. We thank Abilio De Freitas for pointing out several typographical errors.

Note added in proof. After this work was completed, we became aware of a similar calculation [85] that finds complete agreement with the results presented here.

A. Finite two-loop contributions

In this appendix we give explicit formulae the coefficients $A, \ldots, C$ for the finite two-loop amplitudes defined in section 4.4 for the four quark scattering processes of eqs. (4.13)–(4.15).

As usual, the polylogarithms $\mathrm{Li}_n(w)$ are defined by

$$\begin{aligned} \mathrm{Li}_n(w) &= \int_0^w \frac{dt}{t} \mathrm{Li}_{n-1}(t) \qquad \text{for } n = 2, 3, 4 \\ \mathrm{Li}_2(w) &= -\int_0^w \frac{dt}{t} \log(1-t) \,. \end{aligned} \tag{A.1}$$

Using the standard polylogarithm identities [86], we retain the polylogarithms with arguments x, $1-x$ and $(x-1)/x$, where

$$x = -\frac{t}{s}\,, \qquad y = -\frac{u}{s} = 1 - x\,, \qquad z = -\frac{u}{t} = \frac{x-1}{x}\,. \tag{A.2}$$

For convenience, we also introduce the following logarithms

$$X = \log\left(\frac{-t}{s}\right), \qquad Y = \log\left(\frac{-u}{s}\right), \qquad S = \log\left(\frac{s}{\mu^2}\right), \tag{A.3}$$

where μ is the renormalisation scale.

A.1 $q(p_2,+) + \bar{q}(p_1,-) \to Q(p_3,+) + \bar{Q}(p_4,-)$

$$\begin{aligned} A^{(2),[1]}_{s+-+-} = {} & \frac{t^2}{u^2}\Bigg(\frac{3}{2}\mathrm{Li}_4(x) - X\,\mathrm{Li}_3(x) + \frac{1}{4}X^2\mathrm{Li}_2(x) + X\zeta_3 - \frac{1}{4}X^2\pi^2 + \frac{1}{16}X^4 + \\ & \quad + \frac{16}{9}X^2 - \frac{1}{60}\pi^4 - \frac{37}{72}X^3 - \frac{29}{36}X\pi^2\Bigg) + \\ & + \frac{t}{u}\Bigg(\frac{1}{2}\mathrm{Li}_3(x) - \frac{1}{2}X\,\mathrm{Li}_2(x) + \zeta_3 - \frac{29}{36}\pi^2 + \frac{1}{6}X^2 - \frac{2}{3}X\pi^2 + \\ & \quad + \frac{32}{9}X + \frac{1}{3}X^3 - \frac{1}{4}X^2Y\Bigg) + \\ & + \Bigg(-\frac{1}{2}\mathrm{Li}_4(x) - \frac{4}{3}\mathrm{Li}_3(x) + \frac{4}{3}X\,\mathrm{Li}_2(x) - \frac{1}{3}\pi^2\mathrm{Li}_2(x) + \frac{1}{4}X^2\mathrm{Li}_2(x) - \\ & \quad - \frac{7}{2}Y\zeta_3 + 2X\zeta_3 + \frac{269}{72}\zeta_3 + \frac{227}{144}X - \frac{409}{432}Y + \frac{1}{48}X^4 + \frac{137}{1440}\pi^4 - \\ & \quad - \frac{1}{3}XY\pi^2 - \frac{37}{72}X^3 - -\frac{7}{6}X\pi^2 + \frac{2}{3}X^2Y + \frac{23213}{5184} + \frac{1}{6}X^3Y + \frac{28}{9}X^2 - \\ & \quad - \frac{25}{18}\pi^2 - \frac{1}{6}X^2\pi^2 + \frac{11}{48}Y\pi^2\Bigg) + \\ & + i\pi\Bigg(\frac{t^2}{u^2}\Bigg(-\mathrm{Li}_3(x) + \frac{1}{2}X\,\mathrm{Li}_2(x) + \zeta_3 + \frac{32}{9}X - \frac{5}{8}X^2 + \frac{1}{4}X^3\Bigg) + \\ & \quad + \frac{t}{u}\Bigg(-\frac{1}{2}\mathrm{Li}_2(x) + X^2 - \frac{1}{2}XY + \frac{13}{6}X + \frac{32}{9}\Bigg) + \\ & \quad + \Bigg(\frac{1}{2}X\,\mathrm{Li}_2(x) + \frac{4}{3}\mathrm{Li}_2(x) - \frac{3}{2}\zeta_3 + \frac{55}{9} + \frac{4}{3}XY - \frac{1}{6}X\pi^2 + \frac{23}{9}X - \end{aligned}$$

$$-\frac{5}{8}X^2+\frac{1}{12}X^3+\frac{1}{2}X^2Y-\frac{19}{144}\pi^2\Big)\Big) \tag{A.4}$$

$$\begin{aligned}
B^{(2),[1]}_{s+-+-} = {}& \frac{t}{u}\Big(6\mathrm{Li}_4(x)-6\mathrm{Li}_4(y)+6\mathrm{Li}_4(z)-6Y\mathrm{Li}_3(x)-\frac{3}{2}\mathrm{Li}_3(x)-6\mathrm{Li}_3(y)+\\
&+\frac{3}{2}X\mathrm{Li}_2(x)+\pi^2\mathrm{Li}_2(x)+6Y\mathrm{Li}_2(y)+7\zeta_3+6Y\zeta_3-\frac{37}{9}X+\frac{1}{4}Y^4-XY^3+\\
&+\frac{5}{4}X\pi^2+X^2Y-\frac{7}{60}\pi^4+\frac{1}{2}Y^2\pi^2+\frac{17}{12}X^2+\frac{19}{9}\pi^2+\frac{1}{4}XY+3XY^2+\\
&-\frac{7}{6}Y\pi^2-\frac{5}{6}X^3\Big)+\frac{5}{2}\frac{u}{t}Y^2+\\
&+\frac{t^2}{u^2}\Big(\frac{11}{2}\mathrm{Li}_4(x)-8\mathrm{Li}_4(y)+8\mathrm{Li}_4(z)-\mathrm{Li}_3(x)-7Y\mathrm{Li}_3(x)+2X\mathrm{Li}_3(x)+\\
&+2X\mathrm{Li}_3(y)-\frac{3}{4}X^2\mathrm{Li}_2(x)+X\mathrm{Li}_2(x)+\frac{2}{3}\pi^2\mathrm{Li}_2(x)+XY\mathrm{Li}_2(x)+\\
&+7Y\zeta_3+\zeta_3-2X\zeta_3-\frac{5}{6}XY\pi^2+\frac{1}{8}X^2Y+\frac{7}{8}X^2\pi^2+\frac{1}{3}Y^4-\\
&-\frac{14}{9}X^2-\frac{1}{6}X^3Y-\frac{3}{16}X^4-\frac{4}{3}XY^3+\frac{2}{3}Y^2\pi^2+\frac{101}{72}X^3\\
&+X^2Y^2+\frac{13}{9}X\pi^2-\frac{1}{60}\pi^4\Big)+\\
&+\Big(-\frac{5}{2}\mathrm{Li}_4(x)+2\mathrm{Li}_4(y)-2\mathrm{Li}_4(z)+X\mathrm{Li}_3(x)+\frac{19}{6}\mathrm{Li}_3(x)+Y\mathrm{Li}_3(x)-\\
&-2X\mathrm{Li}_3(y)-\frac{29}{3}\mathrm{Li}_3(y)-\frac{3}{4}X^2\mathrm{Li}_2(x)+\frac{4}{3}\pi^2\mathrm{Li}_2(x)-\frac{19}{6}X\mathrm{Li}_2(x)-\\
&-2XY\mathrm{Li}_2(x)+\frac{29}{3}Y\mathrm{Li}_2(y)-XY\mathrm{Li}_2(y)-\frac{3}{2}Y\zeta_3-\frac{11}{72}\zeta_3+\frac{5}{2}X\zeta_3-\\
&-\frac{271}{54}X+\frac{5}{8}X^2\pi^2-\frac{1}{3}X^3Y+\frac{31}{16}X\pi^2+\frac{1}{24}Y^4+\frac{1}{3}XY\pi^2-\\
&-\frac{275}{54}Y-\frac{403}{72}X^2+\frac{89}{72}X^3-\frac{1}{16}X^4-\frac{71}{36}Y^3+\frac{779}{72}Y^2+\frac{1}{3}XY^3-\\
&-\frac{1}{2}Y^2\pi^2-\frac{41}{24}X^2Y+\frac{49}{12}XY^2+\frac{5}{2}XY-\frac{7}{4}X^2Y^2-\frac{475}{144}Y\pi^2+\\
&+\frac{83}{48}\pi^2-\frac{13}{288}\pi^4+\frac{30659}{1296}\Big)+\\
&+i\pi\Big(\frac{t}{u}\Big(-6\mathrm{Li}_3(x)-\frac{9}{2}\mathrm{Li}_2(x)+6\zeta_3-\frac{37}{9}+2XY-\frac{9}{4}X^2-\frac{1}{12}\pi^2-\\
&-\frac{7}{12}X+\frac{1}{4}Y\Big)+5\frac{u}{t}Y+\frac{t^2}{u^2}\times\\
&\times\Big(-5\mathrm{Li}_3(x)+2\mathrm{Li}_3(y)-\frac{1}{2}Y\mathrm{Li}_2(x)-\frac{1}{2}X\mathrm{Li}_2(x)+\mathrm{Li}_2(x)-\\
&-\frac{3}{2}Y\mathrm{Li}_2(y)+5\zeta_3+\frac{1}{12}Y\pi^2-\frac{1}{2}XY^2-\frac{28}{9}X-\frac{3}{4}X^3+\\
&+\frac{1}{12}X\pi^2-\frac{1}{6}\pi^2+2X^2+\frac{1}{4}XY\Big)+\\
&+\Big(2\mathrm{Li}_3(x)-2\mathrm{Li}_3(y)-\frac{77}{6}\mathrm{Li}_2(x)-\frac{5}{2}X\mathrm{Li}_2(x)+\frac{1}{2}Y\mathrm{Li}_2(x)+\frac{3}{2}Y\mathrm{Li}_2(y)+\\
&+\zeta_3-\frac{5}{12}Y\pi^2+\frac{41}{9}-X^2Y-3Y^2-\frac{59}{12}XY-\frac{49}{36}X+\frac{7}{4}X^2-\frac{1}{4}X^3+
\end{aligned}$$

$$
+\frac{1}{2}Y^3+\frac{25}{36}\pi^2+XY^2+\frac{1}{4}X\pi^2+\frac{473}{36}Y\Big)\Big) \tag{A.5}
$$

$$
\begin{aligned}
C^{(2),[1]}_{s+-+-} = \frac{t^2}{u^2}\Big(&-6\mathrm{Li}_4(x)+6\mathrm{Li}_4(y)-6\mathrm{Li}_4(z)+3\mathrm{Li}_3(x)+3Y\mathrm{Li}_3(x)-X\mathrm{Li}_3(x)-\\
&-6X\mathrm{Li}_3(y)+X^2\mathrm{Li}_2(x)-3X\mathrm{Li}_2(x)+\pi^2\mathrm{Li}_2(x)-3XY\mathrm{Li}_2(x)-3Y\zeta_3-\\
&-3\zeta_3+X\zeta_3+\frac{5}{2}XY\pi^2-\frac{3}{8}X^2Y-\frac{23}{24}X^2\pi^2-\frac{1}{4}Y^4-\frac{9}{2}X^2+\frac{1}{2}X^3Y+\\
&+\frac{1}{6}X^4+XY^3-\frac{1}{2}Y^2\pi^2-\frac{23}{24}X^3-3X^2Y^2+\frac{1}{6}X\pi^2-\frac{13}{60}\pi^4\Big)+\\
+\frac{t}{u}\Big(&2\mathrm{Li}_3(x)-2X\mathrm{Li}_2(x)-8\zeta_3-6X+\frac{1}{2}Y\pi^2-\frac{1}{12}X\pi^2-\frac{7}{4}X^2Y-\\
&-\frac{3}{4}XY+\frac{7}{6}\pi^2-\frac{15}{4}X^2+\frac{3}{4}X^3\Big)+\frac{3}{2}\frac{u}{t}Y^2+\\
+\Big(&10\mathrm{Li}_4(x)+2\mathrm{Li}_4(y)+6\mathrm{Li}_4(z)-\mathrm{Li}_3(x)-3Y\mathrm{Li}_3(x)-3X\mathrm{Li}_3(x)+\\
&+6X\mathrm{Li}_3(y)-8Y\mathrm{Li}_3(y)+X^2\mathrm{Li}_2(x)+6XY\mathrm{Li}_2(x)+X\mathrm{Li}_2(x)+\\
&+\frac{1}{3}\pi^2\mathrm{Li}_2(x)+4Y^2\mathrm{Li}_2(y)+3XY\mathrm{Li}_2(y)+6Y\zeta_3-4X\zeta_3-\frac{35}{4}\zeta_3+\\
&+\frac{93}{16}X+\frac{7}{24}Y^4-\frac{9}{4}Y^3-\frac{7}{8}X^2\pi^2+\frac{1}{6}X^3Y+\frac{1}{8}X^2-\frac{13}{24}X^3+\frac{1}{24}X^4+\\
&+\frac{2}{3}XY\pi^2+\frac{71}{8}Y^2+\frac{1}{3}XY^3-\frac{2}{3}Y^2\pi^2+\frac{7}{8}X^2Y+\frac{9}{4}XY^2+\\
&+\frac{21}{4}X^2Y^2-\frac{15}{2}XY+\frac{1}{4}Y\pi^2-\frac{189}{16}Y+\frac{85}{48}\pi^2-\frac{23}{120}\pi^4+\frac{511}{64}\Big)+\\
+i\pi\Big(&\frac{t^2}{u^2}\Big(2\mathrm{Li}_3(x)-6\mathrm{Li}_3(y)+\frac{3}{2}Y\mathrm{Li}_2(x)-X\mathrm{Li}_2(x)-3\mathrm{Li}_2(x)+\\
&\quad+\frac{9}{2}Y\mathrm{Li}_2(y)-2\zeta_3-\frac{1}{4}Y\pi^2+\frac{3}{2}XY^2-9X+\frac{2}{3}X^3-\frac{1}{12}X\pi^2\\
&\quad+\frac{1}{2}\pi^2-\frac{7}{4}X^2-\frac{3}{4}XY\Big)+\\
&+\frac{t}{u}\Big(-2\mathrm{Li}_2(x)+\frac{5}{12}\pi^2-6+\frac{3}{2}X^2-\frac{7}{2}XY-\frac{3}{4}Y-\frac{33}{4}X\Big)+3\frac{u}{t}Y+\\
&+\Big(-6\mathrm{Li}_3(x)-2\mathrm{Li}_3(y)+\mathrm{Li}_2(x)+5X\mathrm{Li}_2(x)+\frac{9}{2}Y\mathrm{Li}_2(x)+\frac{19}{2}Y\mathrm{Li}_2(y)+2\zeta_3-\\
&-\frac{3}{4}Y\pi^2-6+\frac{1}{2}X^2Y-\frac{9}{2}Y^2+\frac{25}{4}XY-\frac{29}{4}X-\frac{5}{4}X^2+\frac{1}{6}X^3+\frac{1}{6}Y^3-\\
&-\frac{1}{12}\pi^2+7XY^2+\frac{1}{12}X\pi^2+\frac{41}{4}Y\Big)\Big)
\end{aligned} \tag{A.6}
$$

$$
\begin{aligned}
D^{(2),[1]}_{s+-+-} = &\frac{t}{u}\Big(-\frac{8}{9}X+\frac{2}{9}\pi^2-\frac{1}{6}X^2\Big)+\frac{t^2}{u^2}\Big(\frac{1}{18}X^3-\frac{4}{9}X^2+\frac{2}{9}X\pi^2\Big)+\\
&+\Big(\frac{1}{3}\mathrm{Li}_3(x)-\frac{1}{3}X\mathrm{Li}_2(x)-\frac{49}{36}\zeta_3+\frac{1}{36}X-\frac{1}{6}X^2Y-\frac{19}{36}X^2+\frac{1}{18}X^3-\frac{455}{108}+\\
&\quad+\frac{19}{24}\pi^2-\frac{1}{24}Y\pi^2+\frac{1}{3}X\pi^2+\frac{25}{108}Y\Big)+\\
&+i\pi\Big(\frac{t}{u}\Big(-\frac{8}{9}-\frac{2}{3}X\Big)-\frac{8}{9}\frac{t^2}{u^2}X+\Big(-\frac{1}{3}\mathrm{Li}_2(x)-\frac{361}{108}-\frac{1}{3}XY+\frac{5}{72}\pi^2-\frac{7}{18}X\Big)\Big)
\end{aligned} \tag{A.7}
$$

$$
\begin{aligned}
E^{(2),[1]}_{s+-+-} = {} & \frac{t^2}{u^2}\left(\frac{8}{9}X^2-\frac{4}{9}X\pi^2-\frac{1}{9}X^3\right)+\frac{t}{u}\left(\frac{1}{3}X^2+\frac{16}{9}X-\frac{4}{9}\pi^2\right)+ \\
& +\Bigg(-\frac{2}{3}\mathrm{Li}_3(x)+\frac{2}{3}\mathrm{Li}_3(y)+\frac{2}{3}X\mathrm{Li}_2(x)-\frac{2}{3}Y\mathrm{Li}_2(y)-\frac{35}{36}\zeta_3+\frac{2}{9}Y^3-\frac{1}{3}XY^2- \\
& \quad -\frac{685}{162}+\frac{77}{72}Y\pi^2-\frac{1}{9}X^3+\frac{223}{108}Y-\frac{13}{24}\pi^2-\frac{5}{8}X\pi^2+\frac{1}{3}X^2Y-\frac{31}{108}X+ \\
& \quad +\frac{19}{18}X^2-\frac{29}{18}Y^2\Bigg)+i\pi\Bigg(\frac{16}{9}\frac{t^2}{u^2}X+\frac{t}{u}\left(\frac{4}{3}X+\frac{16}{9}\right)+ \\
& \qquad +\left(\frac{4}{3}\mathrm{Li}_2(x)-\frac{23}{36}-\frac{11}{9}Y+\frac{7}{9}X+\frac{2}{3}XY-\frac{1}{9}\pi^2\right)\Bigg)
\end{aligned}
\tag{A.8}
$$

$$
F^{(2),[1]}_{s+-+-} = \left(\frac{25}{81}-\frac{1}{9}\pi^2\right)+\frac{10}{27}i\pi \tag{A.9}
$$

$$
\begin{aligned}
A^{(2),[2]}_{s+-+-} = {} & \frac{t^2}{u^2}\Bigg(-5\mathrm{Li}_4(x)+8\mathrm{Li}_4(y)-8\mathrm{Li}_4(z)-\frac{1}{2}\mathrm{Li}_3(x)+7Y\mathrm{Li}_3(x)-2X\mathrm{Li}_3(x) \\
& \quad -2X\mathrm{Li}_3(y)+\frac{1}{2}X^2\mathrm{Li}_2(x)+\frac{1}{2}X\mathrm{Li}_2(x)-\frac{1}{3}\pi^2\mathrm{Li}_2(x)-XY\mathrm{Li}_2(x)- \\
& \quad -7Y\zeta_3+\frac{1}{2}\zeta_3+2X\zeta_3+\frac{5}{6}XY\pi^2-\frac{1}{8}X^2Y+\frac{2}{3}X\pi^2-\frac{1}{3}Y^4-\frac{5}{4}X^2+ \\
& \quad +\frac{1}{6}X^3Y-\frac{2}{3}Y^2\pi^2-\frac{3}{8}X^2\pi^2-X^2Y^2+\frac{4}{3}XY^3-\frac{2}{45}\pi^4\Bigg)+ \\
& +\frac{t}{u}\Bigg(-6\mathrm{Li}_4(x)+6\mathrm{Li}_4(y)-6\mathrm{Li}_4(z)+6Y\mathrm{Li}_3(x)+2\mathrm{Li}_3(x)+6\mathrm{Li}_3(y)- \\
& \quad -2X\mathrm{Li}_2(x)-\pi^2\mathrm{Li}_2(x)-6Y\mathrm{Li}_2(y)-6Y\zeta_3-7\zeta_3+\frac{1}{3}\pi^2+XY^3- \\
& \quad -\frac{1}{4}Y^4+\frac{7}{60}\pi^4-\frac{1}{2}Y^2\pi^2-\frac{1}{12}X\pi^2-\frac{3}{4}X^2Y-\frac{7}{4}XY- \\
& \quad -3XY^2+\frac{5}{6}Y\pi^2-X^2\Bigg)-\frac{3}{2}\frac{u}{t}Y^2+ \\
& +\Bigg(\mathrm{Li}_4(x)-3\mathrm{Li}_4(y)-2\mathrm{Li}_4(z)-2X\mathrm{Li}_3(x)+2Y\mathrm{Li}_3(x)+\mathrm{Li}_3(x)+\frac{28}{3}\mathrm{Li}_3(y)+ \\
& \quad +3Y\mathrm{Li}_3(y)-X\mathrm{Li}_3(y)-X\mathrm{Li}_2(x)+\frac{1}{2}X^2\mathrm{Li}_2(x)-XY\mathrm{Li}_2(x)+\pi^2\mathrm{Li}_2(x)- \\
& \quad -\frac{1}{2}Y^2\mathrm{Li}_2(y)-\frac{28}{3}Y\mathrm{Li}_2(y)-\frac{5}{2}X\zeta_3+2Y\zeta_3-\frac{737}{72}\zeta_3+\frac{13}{6}XY\pi^2-\frac{409}{432}X- \\
& \quad -\frac{1}{4}X^2\pi^2-\frac{1}{8}Y^4+\frac{11}{18}Y^3+\frac{1673}{432}Y-\frac{37}{48}X\pi^2-\frac{385}{72}Y^2+\frac{1}{8}X^2Y-\frac{5}{8}Y^2\pi^2 \\
& \quad -\frac{23213}{5184}-\frac{47}{12}XY^2+\frac{139}{72}Y\pi^2-\frac{13}{8}X^2Y^2+\frac{5}{12}X^3Y-\frac{19}{8}XY+ \\
& \quad +\frac{11}{12}XY^3+\frac{2}{3}\pi^2-\frac{161}{1440}\pi^4\Bigg)+ \\
& +i\pi\Bigg(\frac{t^2}{u^2}\Bigg(5\mathrm{Li}_3(x)-2\mathrm{Li}_3(y)+\frac{1}{2}Y\mathrm{Li}_2(x)+\frac{1}{2}\mathrm{Li}_2(x)+\frac{3}{2}Y\mathrm{Li}_2(y)-5\zeta_3+\frac{1}{12}X\pi^2- \\
& \qquad -\frac{5}{2}X-\frac{1}{12}Y\pi^2-\frac{1}{4}XY+\frac{1}{2}XY^2-\frac{1}{12}\pi^2-\frac{3}{8}X^2+\frac{1}{6}X^3\Bigg)+ \\
& +\frac{t}{u}\left(6\mathrm{Li}_3(x)+4\mathrm{Li}_2(x)-6\zeta_3-\frac{3}{2}XY+\frac{1}{4}X^2+\frac{1}{4}\pi^2-\frac{15}{4}X-\frac{7}{4}Y\right)-3\frac{u}{t}Y+ \\
& +\Bigg(2\mathrm{Li}_3(y)+\frac{25}{3}\mathrm{Li}_2(x)+\frac{1}{2}Y\mathrm{Li}_2(x)+\frac{1}{2}Y\mathrm{Li}_2(y)-\frac{1}{2}\zeta_3-\frac{23}{9}+\frac{1}{2}X\pi^2-
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{2}Y\pi^2+\frac{1}{2}XY^2+\frac{3}{4}Y^2+\frac{1}{4}Y^3-\frac{19}{8}X+\frac{5}{8}X^2+\frac{5}{12}X^3-X^2Y-\\
&-\frac{13}{144}\pi^2+\frac{7}{4}XY-\frac{545}{72}Y\Big)\Big) \qquad \text{(A.10)}
\end{aligned}
$$

$$
\begin{aligned}
B^{(2),[2]}_{s+-+-} = \frac{t^2}{u^2}\Big(&\frac{3}{2}\mathrm{Li}_4(x)-4\mathrm{Li}_4(y)+4\mathrm{Li}_4(z)-\frac{1}{2}\mathrm{Li}_3(x)-2Y\mathrm{Li}_3(x)+2X\mathrm{Li}_3(x)+\\
&+4X\mathrm{Li}_3(y)-\frac{3}{4}X^2\mathrm{Li}_2(x)+\frac{1}{2}X\mathrm{Li}_2(x)-\pi^2\mathrm{Li}_2(x)+2XY\mathrm{Li}_2(x)+2Y\zeta_3+\\
&+\frac{1}{2}\zeta_3-2X\zeta_3--\frac{5}{3}XY\pi^2+\frac{1}{4}X^2Y+\frac{1}{3}X^2\pi^2+\frac{1}{6}Y^4+\frac{127}{36}X^2-\frac{1}{3}X^3Y+\\
&+\frac{1}{48}X^4-\frac{2}{3}XY^3+\frac{1}{3}Y^2\pi^2-\frac{11}{36}X^3+2X^2Y^2-\frac{47}{36}X\pi^2+\frac{41}{180}\pi^4\Big)+\\
+\frac{t}{u}\Big(&-\frac{5}{2}\mathrm{Li}_3(x)+\frac{5}{2}X\mathrm{Li}_2(x)+4\zeta_3-\frac{83}{36}\pi^2+\frac{5}{4}X^2Y+\frac{32}{9}X-\frac{1}{3}X\pi^2+\\
&+\frac{17}{12}X^2-\frac{1}{12}X^3+2XY\Big)-2\frac{u}{t}Y^2+\\
+\Big(&-\frac{9}{2}\mathrm{Li}_4(x)-Y\mathrm{Li}_3(x)-\frac{7}{3}\mathrm{Li}_3(x)+3X\mathrm{Li}_3(x)+\frac{1}{3}\mathrm{Li}_3(y)+2Y\mathrm{Li}_3(y)-\\
&-X\mathrm{Li}_3(y)-2\pi^2\mathrm{Li}_2(x)-\frac{3}{4}X^2\mathrm{Li}_2(x)-XY\mathrm{Li}_2(x)+\frac{7}{3}X\mathrm{Li}_2(x)-2Y^2\mathrm{Li}_2(y)-\\
&-XY\mathrm{Li}_2(y)-\frac{1}{3}Y\mathrm{Li}_2(y)-X\zeta_3+\frac{695}{72}\zeta_3+Y\zeta_3+\frac{1}{6}X\pi^2+\frac{39}{8}XY-\\
&-\frac{5}{2}XY\pi^2+\frac{5}{12}X^2Y+\frac{103}{36}Y^3+\frac{25}{18}X+\frac{3}{8}X^2\pi^2+\frac{29}{36}Y\pi^2-\frac{30659}{1296}-\\
&-\frac{1}{8}Y^4-\frac{193}{18}Y^2+\frac{1}{48}X^4+\frac{13}{8}Y^2\pi^2-\frac{5}{12}X^3Y-\frac{5}{3}XY^2-\frac{17}{12}XY^3-\\
&-\frac{1}{8}X^2Y^2+\frac{49}{6}Y+\frac{125}{72}X^2-\frac{7}{18}X^3-\frac{241}{144}\pi^2+\frac{209}{1440}\pi^4\Big)+\\
+i\pi\Big(&\frac{t^2}{u^2}\Big(4\mathrm{Li}_3(y)+\frac{1}{2}\mathrm{Li}_2(x)+\frac{1}{2}X\mathrm{Li}_2(x)-Y\mathrm{Li}_2(x)-3Y\mathrm{Li}_2(y)+\frac{1}{6}Y\pi^2+\\
&\quad+\frac{1}{2}XY-XY^2+\frac{127}{18}X-\frac{1}{12}X^3-\frac{1}{12}\pi^2-\frac{1}{6}X\pi^2\Big)+\\
&+\frac{t}{u}\Big(\frac{5}{2}\mathrm{Li}_2(x)+2Y+\frac{20}{3}X+\frac{32}{9}+\frac{5}{2}XY-\frac{1}{2}\pi^2-\frac{1}{4}X^2\Big)-4\frac{u}{t}Y+\\
&+\Big(2\mathrm{Li}_3(x)+\mathrm{Li}_3(y)+\frac{8}{3}\mathrm{Li}_2(x)-4Y\mathrm{Li}_2(x)-\frac{3}{2}X\mathrm{Li}_2(x)-8Y\mathrm{Li}_2(y)+\\
&\quad+X^2Y+\frac{17}{12}Y\pi^2-\frac{46}{9}-\frac{7}{12}X\pi^2+\frac{21}{4}Y^2-\frac{11}{12}Y^3+\frac{337}{72}X-X^2-\\
&\quad-\frac{1}{3}X^3-\frac{17}{36}\pi^2-\frac{13}{6}XY-6XY^2-\frac{797}{72}Y\Big)\Big) \qquad \text{(A.11)}
\end{aligned}
$$

$$
\begin{aligned}
C^{(2),[2]}_{s+-+-} = \frac{t^2}{u^2}\Big(&\frac{5}{2}\mathrm{Li}_4(x)-2\mathrm{Li}_4(y)+2\mathrm{Li}_4(z)-\mathrm{Li}_3(x)-Y\mathrm{Li}_3(x)+2X\mathrm{Li}_3(y)-\frac{1}{4}X^2\mathrm{Li}_2(x)+\\
&+XY\mathrm{Li}_2(x)+X\mathrm{Li}_2(x)-\frac{1}{3}\pi^2\mathrm{Li}_2(x)+\zeta_3+Y\zeta_3+\frac{1}{12}Y^4+\frac{1}{8}X^2Y-\\
&-\frac{1}{6}X^3Y-\frac{5}{6}XY\pi^2+2X^2+\frac{3}{8}X^3-\frac{1}{16}X^4-\frac{1}{3}XY^3+\frac{1}{6}Y^2\pi^2+\\
&+\frac{3}{8}X^2\pi^2+X^2Y^2-\frac{1}{6}X\pi^2+\frac{1}{15}\pi^4\Big)+
\end{aligned}
$$

$$
\begin{aligned}
&+\bigg(-\frac{7}{2}\mathrm{Li}_4(x)-\mathrm{Li}_4(y)-2\mathrm{Li}_4(z)+\frac{1}{2}\mathrm{Li}_3(x)+X\mathrm{Li}_3(x)+Y\mathrm{Li}_3(x)+3Y\mathrm{Li}_3(y)-\\
&\quad-2X\mathrm{Li}_3(y)-\frac{1}{3}\pi^2\mathrm{Li}_2(x)-\frac{1}{4}X^2\mathrm{Li}_2(x)-\frac{1}{2}X\mathrm{Li}_2(x)-2XY\mathrm{Li}_2(x)-\\
&\quad-\frac{3}{2}Y^2\mathrm{Li}_2(y)-XY\mathrm{Li}_2(y)-4Y\zeta_3+3X\zeta_3+\frac{23}{4}\zeta_3+\frac{7}{24}X^2\pi^2-\frac{1}{6}X\pi^2-\\
&\quad-\frac{45}{16}X+\frac{5}{2}XY-\frac{1}{3}XY\pi^2+\frac{3}{4}Y^3+\frac{5}{24}X^3-\frac{1}{48}X^4-\frac{511}{64}-\frac{1}{12}Y^4-\frac{29}{8}Y^2-\\
&\quad-\frac{1}{6}XY^3+\frac{1}{6}Y^2\pi^2-\frac{3}{8}X^2Y-\frac{3}{4}XY^2-\frac{7}{4}X^2Y^2+\frac{1}{12}Y\pi^2+\frac{93}{16}Y+\frac{5}{8}X^2-\\
&\quad-\frac{53}{48}\pi^2+\frac{13}{120}\pi^4\bigg)+\\
&+\frac{t}{u}\bigg(-\frac{1}{2}\mathrm{Li}_3(x)+\frac{1}{2}X\mathrm{Li}_2(x)+3\zeta_3+\frac{7}{4}X^2-\frac{1}{6}Y\pi^2-\frac{1}{2}\pi^2+\frac{1}{4}XY+3X\\
&\qquad-\frac{1}{6}X^3-\frac{1}{12}X\pi^2+\frac{1}{2}X^2Y\bigg)-\frac{1}{2}\frac{u}{t}Y^2+\\
&+i\pi\bigg(\frac{t^2}{u^2}\bigg(-\mathrm{Li}_3(x)+2\mathrm{Li}_3(y)-\frac{1}{2}Y\mathrm{Li}_2(x)+\frac{1}{2}X\mathrm{Li}_2(x)+\mathrm{Li}_2(x)-\\
&\qquad\quad-\frac{3}{2}Y\mathrm{Li}_2(y)+\zeta_3+\frac{1}{12}Y\pi^2-\frac{1}{2}XY^2+4X-\frac{1}{4}X^3+\\
&\qquad\quad+\frac{1}{12}X\pi^2-\frac{1}{6}\pi^2+\frac{3}{4}X^2+\frac{1}{4}XY\bigg)+\\
&\qquad+\bigg(2\mathrm{Li}_3(x)+\mathrm{Li}_3(y)-\frac{1}{2}\mathrm{Li}_2(x)-\frac{3}{2}X\mathrm{Li}_2(x)-\frac{3}{2}Y\mathrm{Li}_2(x)-\frac{7}{2}Y\mathrm{Li}_2(y)-\\
&\qquad\quad-\zeta_3+\frac{1}{4}Y\pi^2-\frac{1}{12}X\pi^2+\frac{3}{2}Y^2+\frac{15}{4}X+\frac{1}{2}X^2-\frac{1}{12}X^3+3+\frac{1}{12}\pi^2-\\
&\qquad\quad-\frac{9}{4}XY-\frac{5}{2}XY^2-\frac{19}{4}Y\bigg)+\\
&\qquad+\frac{t}{u}\bigg(\frac{1}{2}\mathrm{Li}_2(x)-\frac{1}{12}\pi^2+\frac{1}{4}Y+\frac{15}{4}X-\frac{1}{4}X^2+3+XY\bigg)-\frac{u}{t}Y\bigg)
\end{aligned}
\tag{A.12}
$$

$$
\begin{aligned}
D^{(2),[2]}_{s+-+-}=&\bigg(-\frac{1}{3}\mathrm{Li}_3(y)+\frac{1}{3}Y\mathrm{Li}_2(y)+\frac{49}{36}\zeta_3+\frac{29}{36}Y^2-\frac{149}{108}Y-\frac{1}{24}X\pi^2+\frac{455}{108}+\\
&\quad+\frac{1}{6}XY^2-\frac{17}{36}Y\pi^2-\frac{41}{72}\pi^2-\frac{1}{9}Y^3+\frac{25}{108}X\bigg)+\\
&+i\pi\bigg(-\frac{1}{3}\mathrm{Li}_2(x)+\frac{11}{18}Y-\frac{1}{72}\pi^2+\frac{265}{108}\bigg)
\end{aligned}
\tag{A.13}
$$

$$
\begin{aligned}
E^{(2),[2]}_{s+-+-}=&\frac{t^2}{u^2}\bigg(\frac{1}{18}X^3-\frac{4}{9}X^2+\frac{2}{9}X\pi^2\bigg)+\frac{t}{u}\bigg(-\frac{8}{9}X+\frac{2}{9}\pi^2-\frac{1}{6}X^2\bigg)+\\
&+\bigg(\frac{1}{3}\mathrm{Li}_3(x)-\frac{1}{3}\mathrm{Li}_3(y)-\frac{1}{3}X\mathrm{Li}_2(x)+\frac{1}{3}Y\mathrm{Li}_2(y)+\frac{35}{36}\zeta_3+\frac{685}{162}+\frac{29}{36}Y^2+\\
&\quad+\frac{1}{3}X\pi^2-\frac{5}{9}Y\pi^2-\frac{19}{36}X^2-\frac{1}{9}Y^3-\frac{11}{12}Y+\frac{23}{72}\pi^2+\frac{1}{18}X^3+\frac{1}{36}X-\\
&\quad-\frac{1}{6}X^2Y+\frac{1}{6}XY^2\bigg)\\
&+i\pi\bigg(-\frac{8}{9}\frac{t^2}{u^2}X+\frac{t}{u}\bigg(-\frac{8}{9}-\frac{2}{3}X\bigg)+
\end{aligned}
$$

$$+\left(-\frac{2}{3}\mathrm{Li}_2(x)+\frac{1}{18}\pi^2-\frac{1}{3}XY+\frac{11}{18}Y+\frac{55}{36}-\frac{7}{18}X\right)\Bigg) \tag{A.14}$$

$$F^{(2),[2]}_{s+-+-} = \left(-\frac{25}{81}+\frac{1}{9}\pi^2\right)-\frac{10}{27}i\pi. \tag{A.15}$$

A.2 $q(p_2,+)+\bar{q}(p_1,-)\to Q(p_3,-)+\bar{Q}(p_4,+)$

$$\begin{aligned}
A^{(2),[1]}_{s+--+} =\ & \frac{3}{2}\frac{t}{u}X^2+\frac{u^2}{t^2}\Bigg(-3\mathrm{Li}_4(x)+3\mathrm{Li}_4(y)-3\mathrm{Li}_4(z)-3X\mathrm{Li}_3(y)-\frac{1}{2}\pi^2\mathrm{Li}_2(x)+3X\zeta_3+\\
&+\frac{1}{2}XY^3-\frac{1}{30}\pi^4-\frac{3}{4}X^2Y^2-\frac{1}{4}Y^2\pi^2-\frac{1}{8}Y^4\Bigg)+\\
&+\frac{u}{t}\Bigg(-3\mathrm{Li}_4(x)+3\mathrm{Li}_4(y)-3\mathrm{Li}_4(z)-3\mathrm{Li}_3(x)-3X\mathrm{Li}_3(y)+3X\mathrm{Li}_2(x)-\\
&-\frac{1}{2}\pi^2\mathrm{Li}_2(x)+3X\zeta_3+3\zeta_3-\frac{1}{8}Y^4-\frac{3}{4}X^2Y^2+\frac{3}{2}X^2Y-\frac{1}{2}X\pi^2+\\
&+\frac{1}{2}\pi^2+\frac{1}{2}XY^3-\frac{1}{30}\pi^4-\frac{1}{4}Y^2\pi^2\Bigg)+\\
&+\Bigg(\mathrm{Li}_4(x)-\frac{29}{6}\mathrm{Li}_3(x)-X\mathrm{Li}_3(x)+\frac{29}{6}X\mathrm{Li}_2(x)+\frac{1}{2}X^2\mathrm{Li}_2(x)-\frac{1}{3}\pi^2\mathrm{Li}_2(x)-\\
&-\frac{7}{2}Y\zeta_3+3X\zeta_3+\frac{413}{72}\zeta_3-\frac{95}{48}X-\frac{409}{432}Y+\frac{1}{12}X^4+\frac{113}{1440}\pi^4-\frac{49}{36}X^3-\\
&-\frac{65}{36}X\pi^2+\frac{29}{12}X^2Y+\frac{23213}{5184}+\frac{1}{6}X^3Y+\frac{56}{9}X^2-\frac{1}{12}\pi^2-\\
&-\frac{5}{12}X^2\pi^2-\frac{1}{3}XY\pi^2+\frac{11}{48}Y\pi^2\Bigg)+\\
&+i\pi\Bigg(3\frac{t}{u}X+\frac{u^2}{t^2}\Big(-3\mathrm{Li}_3(y)+3\zeta_3\Big)+\\
&+\frac{u}{t}\Big(-3\mathrm{Li}_3(y)+3\mathrm{Li}_2(x)+3\zeta_3+3XY-\frac{1}{2}\pi^2\Big)\\
&+\Big(-\mathrm{Li}_3(x)+X\mathrm{Li}_2(x)+\frac{29}{6}\mathrm{Li}_2(x)-\frac{1}{2}\zeta_3+\frac{23}{9}-\frac{1}{6}X\pi^2+\frac{125}{18}X+\\
&+\frac{29}{6}XY+\frac{1}{3}X^3+\frac{1}{2}X^2Y-\frac{91}{144}\pi^2-\frac{9}{4}X^2\Big)\Bigg)
\end{aligned} \tag{A.16}$$

$$\begin{aligned}
B^{(2),[1]}_{s+--+} =\ & \frac{u^2}{t^2}\Bigg(4\mathrm{Li}_4(x)-2\mathrm{Li}_4(y)+4\mathrm{Li}_4(z)+2Y\mathrm{Li}_3(x)-Y\mathrm{Li}_3(y)-\mathrm{Li}_3(y)+5X\mathrm{Li}_3(y)+\\
&+\frac{4}{3}\pi^2\mathrm{Li}_2(x)-\frac{3}{2}XY\mathrm{Li}_2(x)+Y\mathrm{Li}_2(y)-\frac{1}{2}XY\mathrm{Li}_2(y)-5X\zeta_3+\zeta_3+Y\zeta_3+\\
&+\frac{73}{18}Y^2-\frac{5}{6}XY^3+\frac{1}{4}Y^4+\frac{1}{24}Y^2\pi^2-\frac{59}{72}Y^3+\frac{1}{2}X^2Y^2+\frac{1}{45}\pi^4+\frac{1}{8}XY^2-\\
&-\frac{13}{9}Y\pi^2+\frac{1}{12}XY\pi^2\Bigg)-\\
&-\frac{7}{2}\frac{t}{u}X^2+\frac{u}{t}\Bigg(6\mathrm{Li}_4(x)-6\mathrm{Li}_4(y)+6\mathrm{Li}_4(z)+6\mathrm{Li}_3(x)+6X\mathrm{Li}_3(y)+\pi^2\mathrm{Li}_2(x)-\\
&-6X\mathrm{Li}_2(x)-2\zeta_3-6X\zeta_3+\frac{3}{2}X^2Y^2+\frac{1}{4}Y^4+\frac{1}{15}\pi^4+\frac{7}{12}Y^2-\\
&-XY^3+\frac{64}{9}Y+\frac{5}{6}X\pi^2-3X^2Y+\frac{1}{2}Y^2\pi^2+\frac{1}{4}Y^3-\frac{25}{9}\pi^2+
\end{aligned}$$

$$+\frac{1}{4}XY+\frac{1}{4}XY^2-\frac{13}{12}Y\pi^2\Big)+$$
$$+\Big(-\mathrm{Li}_4(x)-4\mathrm{Li}_4(y)+2\mathrm{Li}_4(z)+3X\mathrm{Li}_3(x)+\frac{29}{3}\mathrm{Li}_3(x)-2Y\mathrm{Li}_3(x)+$$
$$+X\mathrm{Li}_3(y)-\frac{8}{3}\mathrm{Li}_3(y)+Y\mathrm{Li}_3(y)-\frac{3}{2}X^2\mathrm{Li}_2(x)+\frac{2}{3}\pi^2\mathrm{Li}_2(x)-\frac{29}{3}X\mathrm{Li}_2(x)+$$
$$+\frac{1}{2}XY\mathrm{Li}_2(x)-\frac{1}{2}XY\mathrm{Li}_2(y)+\frac{8}{3}Y\mathrm{Li}_2(y)-\frac{3}{2}Y\zeta_3-\frac{659}{72}\zeta_3-\frac{1}{2}X\zeta_3+$$
$$+\frac{3}{2}X^2\pi^2-\frac{49}{54}X+\frac{427}{144}X\pi^2+\frac{1}{8}Y^4-\frac{7}{12}XY\pi^2-\frac{1}{2}X^3Y-\frac{869}{72}X^2+$$
$$+\frac{125}{36}X^3-\frac{1}{4}X^4-\frac{65}{72}Y^3+\frac{349}{72}Y^2-\frac{1}{6}XY^3+\frac{1}{8}Y^2\pi^2-\frac{67}{12}X^2Y+$$
$$+\frac{29}{24}XY^2+\frac{5}{2}XY+\frac{1}{4}X^2Y^2-\frac{85}{48}Y\pi^2+\frac{109}{54}Y-\frac{455}{144}\pi^2+$$
$$+\frac{143}{1440}\pi^4+\frac{30659}{1296}\Big)+$$
$$+i\pi\Bigg(\frac{u^2}{t^2}\Big(2\mathrm{Li}_3(x)+4\mathrm{Li}_3(y)-\mathrm{Li}_2(x)+\frac{1}{2}Y\mathrm{Li}_2(x)-X\mathrm{Li}_2(x)+\frac{3}{2}Y\mathrm{Li}_2(y)-4\zeta_3-$$
$$-\frac{1}{6}Y\pi^2+\frac{1}{3}Y^3-\frac{3}{4}XY+\frac{1}{2}XY^2-Y^2+\frac{73}{9}Y\Big)-7\frac{t}{u}X+$$
$$+\frac{u}{t}\Big(6\mathrm{Li}_3(y)-6\mathrm{Li}_2(x)-6\zeta_3+\frac{64}{9}-\frac{11}{2}XY+\frac{1}{4}X+\frac{3}{4}\pi^2+Y^2+\frac{61}{12}Y\Big)+$$
$$+\Big(\mathrm{Li}_3(x)+2\mathrm{Li}_3(y)-\frac{37}{3}\mathrm{Li}_2(x)-2X\mathrm{Li}_2(x)-\frac{1}{2}Y\mathrm{Li}_2(x)-\frac{3}{2}Y\mathrm{Li}_2(y)-$$
$$-2\zeta_3-\frac{1}{6}Y\pi^2+\frac{142}{9}-Y^2+\frac{1}{6}Y^3-\frac{383}{36}X+6X^2-X^3-X^2Y+\frac{49}{36}\pi^2-$$
$$-\frac{137}{12}XY+\frac{1}{3}X\pi^2+\frac{175}{36}Y\Big)\Bigg) \tag{A.17}$$

$$C^{(2),[1]}_{s+--+}=\frac{u^2}{t^2}\Big(6\mathrm{Li}_4(x)+6\mathrm{Li}_4(z)-6Y\mathrm{Li}_3(x)+3X\mathrm{Li}_3(y)-5Y\mathrm{Li}_3(y)+3\mathrm{Li}_3(y)+$$
$$+\frac{9}{2}XY\mathrm{Li}_2(x)-\pi^2\mathrm{Li}_2(x)+2Y^2\mathrm{Li}_2(y)+\frac{3}{2}XY\mathrm{Li}_2(y)-3Y\mathrm{Li}_2(y)+$$
$$+5Y\zeta_3-3\zeta_3-3X\zeta_3+3X^2Y^2+\frac{3}{2}Y^2+\frac{1}{3}Y^4+\frac{5}{24}Y^2\pi^2-\frac{1}{2}XY^3-$$
$$-\frac{7}{24}Y^3-\frac{3}{8}XY^2-\frac{7}{6}Y\pi^2-\frac{1}{4}XY\pi^2\Big)+$$
$$+\frac{u}{t}\Big(4\mathrm{Li}_3(y)-4Y\mathrm{Li}_2(y)-4\zeta_3-\frac{17}{12}Y\pi^2+\frac{1}{2}X\pi^2+6Y-\frac{11}{4}XY^2-\frac{3}{4}XY$$
$$+\frac{7}{4}Y^3+\frac{9}{4}Y^2-\frac{1}{6}\pi^2\Big)+\frac{3}{2}\frac{t}{u}X^2+$$
$$+\Big(-2\mathrm{Li}_4(x)+8\mathrm{Li}_4(y)-6\mathrm{Li}_4(z)+6Y\mathrm{Li}_3(x)-4X\mathrm{Li}_3(x)-3X\mathrm{Li}_3(y)-$$
$$-3Y\mathrm{Li}_3(y)+\mathrm{Li}_3(y)-\frac{3}{2}XY\mathrm{Li}_2(x)+2X^2\mathrm{Li}_2(x)+\frac{7}{3}\pi^2\mathrm{Li}_2(x)-Y\mathrm{Li}_2(y)+$$
$$+2Y^2\mathrm{Li}_2(y)+\frac{3}{2}XY\mathrm{Li}_2(y)-2Y\zeta_3-\frac{19}{4}\zeta_3-\frac{5}{24}Y^3-\frac{7}{24}Y^4+\frac{189}{16}X-$$
$$-\frac{15}{2}XY+\frac{2}{3}X^3Y+\frac{41}{12}XY\pi^2+\frac{3}{4}X\pi^2+\frac{5}{24}X^4-\frac{11}{6}X^2\pi^2+\frac{65}{8}Y^2-$$

$$
\begin{aligned}
&-\frac{9}{4}X^3-\frac{11}{8}Y^2\pi^2+\frac{9}{4}X^2Y-\frac{1}{8}XY^2+\frac{11}{6}XY^3-\frac{3}{4}X^2Y^2-\frac{1}{2}Y\pi^2-\\
&-\frac{93}{16}Y+\frac{7}{8}X^2+\frac{7}{16}\pi^2-\frac{49}{120}\pi^4+\frac{511}{64}\Bigg)+\\
&+i\pi\Bigg(\frac{u^2}{t^2}\bigg(-6\mathrm{Li}_3(x)-2\mathrm{Li}_3(y)+3\mathrm{Li}_2(x)+\frac{3}{2}Y\mathrm{Li}_2(x)+3X\mathrm{Li}_2(x)+\frac{5}{2}Y\mathrm{Li}_2(y)+\\
&\qquad+2\zeta_3+\frac{1}{3}Y\pi^2+\frac{1}{3}Y^3+\frac{9}{4}XY+\frac{3}{2}XY^2+\frac{1}{4}Y^2+3Y\bigg)+\\
&\quad+\frac{u}{t}\bigg(4\mathrm{Li}_2(x)+\frac{5}{12}\pi^2-\frac{3}{4}X-\frac{3}{2}XY+\frac{9}{2}Y^2+\frac{15}{4}Y+6\bigg)+3\frac{t}{u}X+\\
&\quad+\bigg(2\mathrm{Li}_3(x)-6\mathrm{Li}_3(y)+\mathrm{Li}_2(x)+X\mathrm{Li}_2(x)+\frac{9}{2}Y\mathrm{Li}_2(x)+\frac{23}{2}Y\mathrm{Li}_2(y)-\\
&\qquad-2\zeta_3-\frac{4}{3}Y\pi^2+6-\frac{1}{4}Y^2-\frac{1}{6}Y^3-\frac{23}{4}X-\frac{9}{2}X^2+\frac{5}{6}X^3+\frac{1}{2}X^2Y+\\
&\qquad+\frac{5}{12}\pi^2+\frac{21}{4}XY+7XY^2+\frac{35}{4}Y\bigg)\Bigg)
\end{aligned}
\tag{A.18}
$$

$$
\begin{aligned}
D^{(2),[1]}_{s+--+} = &\bigg(\frac{1}{3}\mathrm{Li}_3(x)-\frac{1}{3}X\mathrm{Li}_2(x)-\frac{49}{36}\zeta_3+\frac{5}{9}X\pi^2-\frac{1}{6}X^2Y+\frac{1}{9}X^3+\frac{11}{12}X-\\
&-\frac{455}{108}-\frac{29}{36}X^2-\frac{1}{24}Y\pi^2+\frac{25}{108}Y+\frac{41}{72}\pi^2\bigg)+\\
&+i\pi\bigg(-\frac{1}{3}\mathrm{Li}_2(x)-\frac{265}{108}-\frac{1}{3}XY+\frac{5}{72}\pi^2-\frac{11}{18}X\bigg)
\end{aligned}
\tag{A.19}
$$

$$
\begin{aligned}
E^{(2),[1]}_{s+--+} = &\frac{u^2}{t^2}\bigg(-\frac{8}{9}Y^2+\frac{4}{9}Y\pi^2+\frac{1}{9}Y^3\bigg)+\frac{u}{t}\bigg(\frac{4}{9}\pi^2-\frac{1}{3}Y^2-\frac{16}{9}Y\bigg)+\\
&+\bigg(-\frac{2}{3}\mathrm{Li}_3(x)+\frac{2}{3}\mathrm{Li}_3(y)+\frac{2}{3}X\mathrm{Li}_2(x)-\frac{2}{3}Y\mathrm{Li}_2(y)-\frac{35}{36}\zeta_3+\frac{29}{18}X^2+\frac{5}{8}Y\pi^2-\\
&\quad-\frac{77}{72}X\pi^2+\frac{1}{3}X^2Y-\frac{19}{18}Y^2-\frac{2}{9}X^3-\frac{223}{108}X+\frac{25}{72}\pi^2-\frac{685}{162}-\frac{1}{3}XY^2+\\
&\quad+\frac{31}{108}Y+\frac{1}{9}Y^3\bigg)+\\
&+i\pi\bigg(-\frac{16}{9}\frac{u^2}{t^2}Y+\frac{u}{t}\bigg(-\frac{16}{9}-\frac{4}{3}Y\bigg)+\\
&\quad+\bigg(\frac{4}{3}\mathrm{Li}_2(x)-\frac{151}{36}+\frac{2}{3}XY+\frac{11}{9}X-\frac{1}{9}\pi^2-\frac{7}{9}Y\bigg)\bigg)
\end{aligned}
\tag{A.20}
$$

$$
F^{(2),[1]}_{s+--+} = \bigg(\frac{25}{81}-\frac{1}{9}\pi^2\bigg)+\frac{10}{27}i\pi
\tag{A.21}
$$

$$
\begin{aligned}
A^{(2),[2]}_{s+--+} = &\frac{u^2}{t^2}\bigg(-3\mathrm{Li}_4(x)-\frac{1}{2}\mathrm{Li}_4(y)-3\mathrm{Li}_4(z)+2Y\mathrm{Li}_3(y)+\frac{3}{2}\mathrm{Li}_3(y)-3X\mathrm{Li}_3(y)-\\
&\quad-\frac{1}{6}\pi^2\mathrm{Li}_2(x)-\frac{1}{4}Y^2\mathrm{Li}_2(y)-\frac{3}{2}Y\mathrm{Li}_2(y)-2Y\zeta_3+3X\zeta_3-\frac{3}{2}\zeta_3-\\
&\quad-\frac{3}{4}X^2Y^2+\frac{1}{3}XY\pi^2-\frac{7}{48}Y^4+\frac{1}{180}\pi^4-\frac{37}{36}Y^2+\frac{1}{2}XY^3+\frac{11}{36}Y^3-\\
&\quad-\frac{1}{36}Y\pi^2+\frac{1}{6}Y^2\pi^2\bigg)+\frac{1}{2}\frac{t}{u}X^2+\\
&+\frac{u}{t}\bigg(-3\mathrm{Li}_4(x)+3\mathrm{Li}_4(y)-3\mathrm{Li}_4(z)-3\mathrm{Li}_3(x)-\frac{3}{2}\mathrm{Li}_3(y)-3X\mathrm{Li}_3(y)-
\end{aligned}
$$

JHEP04(2004)021

$$
\begin{aligned}
&-\frac{1}{2}\pi^2\mathrm{Li}_2(x)+3X\mathrm{Li}_2(x)+\frac{3}{2}Y\mathrm{Li}_2(y)+\zeta_3+3X\zeta_3+\frac{1}{12}Y^3+\frac{3}{2}X^2Y-\\
&-\frac{1}{8}Y^4-\frac{1}{6}X\pi^2-\frac{32}{9}Y+\frac{7}{12}Y^2+\frac{1}{2}XY^3+\frac{1}{2}Y\pi^2+\frac{1}{4}XY^2+\frac{3}{2}XY-\\
&-\frac{3}{4}X^2Y^2-\frac{1}{4}Y^2\pi^2+\frac{5}{36}\pi^2-\frac{1}{30}\pi^4\Bigg)+\\
&+\Bigg(2\mathrm{Li}_4(x)+\frac{5}{2}\mathrm{Li}_4(y)-4\mathrm{Li}_4(z)+3Y\mathrm{Li}_3(x)-\frac{9}{2}\mathrm{Li}_3(x)-4X\mathrm{Li}_3(x)+Y\mathrm{Li}_3(y)+\\
&+\frac{1}{3}\mathrm{Li}_3(y)-3X\mathrm{Li}_3(y)+\frac{9}{2}X\mathrm{Li}_2(x)+\frac{4}{3}\pi^2\mathrm{Li}_2(x)-2XY\mathrm{Li}_2(x)+X^2\mathrm{Li}_2(x)-\\
&-\frac{1}{4}Y^2\mathrm{Li}_2(y)-\frac{1}{3}Y\mathrm{Li}_2(y)+3Y\zeta_3-\frac{1}{2}X\zeta_3-\frac{17}{72}\zeta_3-\frac{409}{432}X+\frac{7}{12}X^3Y-\\
&-\frac{5}{8}X^2\pi^2-\frac{3}{16}Y^4+\frac{7}{18}Y^3+\frac{137}{432}Y+\frac{3}{4}X^2-\frac{23213}{5184}+\frac{8}{3}XY\pi^2+\frac{5}{4}XY^3-\\
&-\frac{29}{24}Y^2\pi^2-\frac{3}{16}X\pi^2-\frac{125}{72}Y^2+\frac{9}{4}X^2Y-\frac{21}{8}X^2Y^2-\frac{1}{8}XY-\frac{2}{3}XY^2+\\
&+\frac{13}{8}Y\pi^2+\frac{35}{36}\pi^2-\frac{449}{1440}\pi^4\Bigg)+\\
&+i\pi\Bigg(\frac{u^2}{t^2}\Bigg(-\mathrm{Li}_3(y)+\frac{3}{2}\mathrm{Li}_2(x)-\frac{1}{2}Y\mathrm{Li}_2(y)+\zeta_3+\frac{3}{4}Y^2-\frac{1}{4}Y^3+\frac{3}{2}XY-\frac{37}{18}Y\Bigg)+\\
&\quad+\frac{t}{u}X+\frac{u}{t}\Bigg(-3\mathrm{Li}_3(y)+\frac{3}{2}\mathrm{Li}_2(x)+3\zeta_3-\frac{32}{9}+2XY+\\
&\qquad\qquad+\frac{3}{2}X-\frac{1}{4}\pi^2-\frac{1}{4}Y^2+\frac{5}{6}Y\Bigg)+\\
&\quad+\Bigg(-\mathrm{Li}_3(x)-2\mathrm{Li}_3(y)+Y\mathrm{Li}_2(x)+\frac{29}{6}\mathrm{Li}_2(x)+\frac{5}{2}Y\mathrm{Li}_2(y)+\frac{5}{2}\zeta_3-\\
&\quad-X^2Y+\frac{7}{12}X\pi^2-\frac{7}{12}Y\pi^2+XY^2-\frac{1}{4}Y^2+\frac{1}{2}Y^3+\frac{7}{12}X^3-\\
&\quad-\frac{55}{9}-\frac{97}{144}\pi^2+\frac{11}{8}X+\frac{7}{2}XY+\frac{5}{72}Y\Bigg)\Bigg)
\end{aligned}
\tag{A.22}
$$

$$
\begin{aligned}
B^{(2),[2]}_{s+--+} = &\Bigg(-\mathrm{Li}_4(x)-4\mathrm{Li}_4(y)+6\mathrm{Li}_4(z)+5X\mathrm{Li}_3(x)-\frac{1}{3}\mathrm{Li}_3(x)-5Y\mathrm{Li}_3(x)+\\
&+4X\mathrm{Li}_3(y)+\frac{11}{6}\mathrm{Li}_3(y)-\frac{3}{2}X^2\mathrm{Li}_2(x)+\frac{1}{3}X\mathrm{Li}_2(x)-3\pi^2\mathrm{Li}_2(x)+\frac{5}{2}XY\mathrm{Li}_2(x)-\\
&-\frac{11}{6}Y\mathrm{Li}_2(y)-\frac{1}{2}XY\mathrm{Li}_2(y)-Y^2\mathrm{Li}_2(y)-4X\zeta_3+\frac{335}{72}\zeta_3+5Y\zeta_3+\frac{17}{24}X^2\pi^2-\\
&-\frac{47}{12}XY\pi^2+\frac{49}{72}Y^3-\frac{47}{36}X\pi^2+\frac{1}{4}Y^4-\frac{7}{12}X^2Y+\frac{21}{8}XY-\frac{11}{18}X^3+\frac{1}{12}Y\pi^2-\\
&-\frac{30659}{1296}-\frac{13}{6}X-\frac{283}{36}Y^2+\frac{1}{24}X^4+2Y^2\pi^2-\frac{3}{4}X^3Y-\frac{13}{24}XY^2-\frac{9}{4}XY^3+\\
&+\frac{23}{8}X^2Y^2+\frac{29}{18}Y+\frac{277}{72}X^2+\frac{109}{48}\pi^2+\frac{179}{480}\pi^4\Bigg)+\\
&+\frac{u^2}{t^2}\Bigg(-2\mathrm{Li}_4(x)-2\mathrm{Li}_4(z)+2Y\mathrm{Li}_3(x)+2Y\mathrm{Li}_3(y)-X\mathrm{Li}_3(y)-\frac{5}{2}\mathrm{Li}_3(y)-\\
&\quad-\frac{3}{2}XY\mathrm{Li}_2(x)-Y^2\mathrm{Li}_2(y)-\frac{1}{2}XY\mathrm{Li}_2(y)+\frac{5}{2}Y\mathrm{Li}_2(y)-2Y\zeta_3+\frac{5}{2}\zeta_3+\\
&\quad+X\zeta_3-X^2Y^2-\frac{127}{36}Y^2+\frac{55}{72}Y^3-\frac{5}{24}Y^4-\frac{1}{24}Y^2\pi^2-\frac{1}{4}XY\pi^2+
\end{aligned}
$$

$$+\frac{1}{8}XY^2+\frac{77}{36}Y\pi^2++\frac{1}{6}XY^3\Big)+\frac{1}{2}\frac{t}{u}X^2+$$
$$+\frac{u}{t}\Big(-\mathrm{Li}_3(y)+Y\mathrm{Li}_2(y)+\zeta_3+\frac{5}{4}XY^2-\frac{5}{4}XY-\frac{59}{9}Y-\frac{13}{6}Y^2+$$
$$+\frac{17}{12}Y\pi^2-\frac{17}{12}Y^3+\frac{77}{36}\pi^2-\frac{1}{2}X\pi^2\Big)+$$
$$+i\pi\Big(\Big(4\mathrm{Li}_3(y)+\frac{13}{6}\mathrm{Li}_2(x)-\frac{7}{2}Y\mathrm{Li}_2(x)-\frac{17}{2}Y\mathrm{Li}_2(y)+\zeta_3-\frac{137}{9}-\frac{3}{4}X\pi^2+$$
$$+\frac{19}{12}Y\pi^2-\frac{11}{2}XY^2+\frac{3}{2}Y^2-\frac{7}{12}Y^3+\frac{347}{72}X-\frac{3}{4}X^2-\frac{5}{12}X^3+X^2Y-$$
$$-\frac{11}{36}\pi^2-\frac{5}{12}XY-\frac{679}{72}Y\Big)+$$
$$+\frac{u^2}{t^2}\Big(2\mathrm{Li}_3(x)+\mathrm{Li}_3(y)-\frac{5}{2}\mathrm{Li}_2(x)-X\mathrm{Li}_2(x)-\frac{3}{2}Y\mathrm{Li}_2(x)-\frac{5}{2}Y\mathrm{Li}_2(y)-$$
$$-\zeta_3-\frac{3}{2}XY^2-\frac{1}{3}Y^3-\frac{9}{4}XY+\frac{1}{4}Y^2-\frac{127}{18}Y\Big)+\frac{t}{u}X+$$
$$+\frac{u}{t}\Big(-\mathrm{Li}_2(x)+\frac{3}{2}XY-\frac{1}{4}\pi^2-\frac{59}{9}-\frac{89}{12}Y-\frac{5}{4}X-\frac{7}{2}Y^2\Big)\Big)\qquad\text{(A.23)}$$
$$C^{(2),[2]}_{s+--+}=\frac{u^2}{t^2}\Big(-2\mathrm{Li}_4(x)-\frac{1}{2}\mathrm{Li}_4(y)-2\mathrm{Li}_4(z)+2Y\mathrm{Li}_3(x)-\mathrm{Li}_3(y)-X\mathrm{Li}_3(y)+2Y\mathrm{Li}_3(y)+$$
$$+\frac{1}{3}\pi^2\mathrm{Li}_2(x)-\frac{3}{2}XY\mathrm{Li}_2(x)+Y\mathrm{Li}_2(y)-\frac{1}{2}XY\mathrm{Li}_2(y)-\frac{3}{4}Y^2\mathrm{Li}_2(y)+\zeta_3+$$
$$+X\zeta_3-2Y\zeta_3+\frac{1}{2}Y\pi^2-\frac{1}{8}Y^2\pi^2+\frac{1}{24}Y^3-Y^2+\frac{1}{12}XY\pi^2-X^2Y^2-$$
$$-\frac{5}{48}Y^4+\frac{1}{8}XY^2+\frac{1}{6}XY^3+\frac{1}{180}\pi^4\Big)-\frac{1}{2}\frac{t}{u}X^2$$
$$+\frac{u}{t}\Big(-\frac{3}{2}\mathrm{Li}_3(y)+\frac{3}{2}Y\mathrm{Li}_2(y)+\zeta_3-\frac{1}{6}X\pi^2-\frac{2}{3}Y^3-3Y+\frac{7}{12}Y\pi^2-$$
$$-\frac{5}{4}Y^2+\frac{1}{4}XY+\frac{1}{6}\pi^2+XY^2\Big)$$
$$+\Big(\mathrm{Li}_4(x)-\frac{5}{2}\mathrm{Li}_4(y)+2\mathrm{Li}_4(z)+X\mathrm{Li}_3(x)-2Y\mathrm{Li}_3(x)-\frac{1}{2}\mathrm{Li}_3(y)+X\mathrm{Li}_3(y)+$$
$$+Y\mathrm{Li}_3(y)-\pi^2\mathrm{Li}_2(x)-\frac{1}{2}X^2\mathrm{Li}_2(x)+\frac{1}{2}XY\mathrm{Li}_2(x)+\frac{1}{2}Y\mathrm{Li}_2(y)-$$
$$-\frac{1}{2}XY\mathrm{Li}_2(y)-\frac{3}{4}Y^2\mathrm{Li}_2(y)+\frac{15}{4}\zeta_3+2X\zeta_3-Y\zeta_3+\frac{2}{3}X^2\pi^2-\frac{3}{4}X^2Y-$$
$$-\frac{5}{12}X\pi^2+\frac{1}{3}Y\pi^2-\frac{5}{4}XY\pi^2+\frac{1}{24}Y^3-\frac{93}{16}X-\frac{1}{12}X^4-\frac{511}{64}+\frac{5}{48}Y^4-$$
$$-\frac{27}{8}Y^2-\frac{2}{3}XY^3+\frac{11}{24}Y^2\pi^2-\frac{1}{6}X^3Y+\frac{1}{8}XY^2+\frac{5}{2}XY+\frac{1}{4}X^2Y^2+$$
$$+\frac{45}{16}Y+\frac{3}{8}X^2+\frac{3}{4}X^3-\frac{7}{16}\pi^2+\frac{61}{360}\pi^4\Big)+$$
$$+i\pi\Big(\frac{u^2}{t^2}\Big(2\mathrm{Li}_3(x)+\mathrm{Li}_3(y)-\mathrm{Li}_2(x)-\frac{1}{2}Y\mathrm{Li}_2(x)-X\mathrm{Li}_2(x)-Y\mathrm{Li}_2(y)-\zeta_3-$$
$$-\frac{1}{6}Y\pi^2-\frac{1}{12}Y^3-\frac{3}{4}XY-\frac{1}{2}XY^2-\frac{1}{4}Y^2-2Y\Big)-\frac{t}{u}X+$$
$$+\frac{u}{t}\Big(-\frac{3}{2}\mathrm{Li}_2(x)+\frac{1}{2}XY-\frac{1}{6}\pi^2-3-\frac{7}{4}Y^2-\frac{9}{4}Y+\frac{1}{4}X\Big)+$$

$$
+\left(-\mathrm{Li}_3(x)+2\mathrm{Li}_3(y)-\frac{1}{2}\mathrm{Li}_2(x)-\frac{3}{2}Y\mathrm{Li}_2(x)-4Y\mathrm{Li}_2(y)+\zeta_3\right.
$$
$$
+\frac{1}{2}Y\pi^2-\frac{5}{2}XY^2+\frac{13}{4}X+\frac{3}{2}X^2-\frac{7}{4}XY+\frac{1}{12}Y^3-
$$
$$
\left.\left.-\frac{1}{6}\pi^2-3-\frac{1}{3}X^3-\frac{17}{4}Y\right)\right) \tag{A.24}
$$

$$
\begin{aligned}
D^{(2),[2]}_{s+--+} = {}& \frac{u^2}{t^2}\left(-\frac{2}{9}Y\pi^2-\frac{1}{18}Y^3+\frac{4}{9}Y^2\right)+\\
&+\left(-\frac{1}{3}\mathrm{Li}_3(y)+\frac{1}{3}Y\mathrm{Li}_2(y)+\frac{49}{36}\zeta_3-\frac{1}{24}X\pi^2-\frac{19}{24}\pi^2+\frac{19}{36}Y^2-\frac{1}{4}Y\pi^2+\right.\\
&\quad\left.+\frac{455}{108}+\frac{1}{6}XY^2+\frac{25}{108}X-\frac{1}{18}Y^3-\frac{53}{108}Y\right)+\frac{u}{t}\left(\frac{1}{6}Y^2-\frac{2}{9}\pi^2+\frac{8}{9}Y\right)+\\
&+i\pi\left(\frac{8}{9}\frac{u^2}{t^2}Y+\left(-\frac{1}{3}\mathrm{Li}_2(x)+\frac{361}{108}+\frac{7}{18}Y-\frac{1}{72}\pi^2\right)+\frac{u}{t}\left(\frac{2}{3}Y+\frac{8}{9}\right)\right)
\end{aligned} \tag{A.25}
$$

$$
\begin{aligned}
E^{(2),[2]}_{s+--+} = {}& \left(\frac{1}{3}\mathrm{Li}_3(x)-\frac{1}{3}\mathrm{Li}_3(y)-\frac{1}{3}X\mathrm{Li}_2(x)+\frac{1}{3}Y\mathrm{Li}_2(y)+\frac{35}{36}\zeta_3-\frac{1}{8}\pi^2+\frac{11}{12}X+\frac{5}{9}X\pi^2\right.\\
&\left.-\frac{1}{18}Y^3-\frac{1}{6}X^2Y+\frac{1}{9}X^3+\frac{19}{36}Y^2-\frac{1}{36}Y+\frac{685}{162}+\frac{1}{6}XY^2-\frac{29}{36}X^2-\frac{1}{3}Y\pi^2\right)+\\
&+\frac{u^2}{t^2}\left(-\frac{2}{9}Y\pi^2-\frac{1}{18}Y^3+\frac{4}{9}Y^2\right)+\frac{u}{t}\left(\frac{1}{6}Y^2-\frac{2}{9}\pi^2+\frac{8}{9}Y\right)+\\
&+i\pi\left(\left(-\frac{2}{3}\mathrm{Li}_2(x)+\frac{119}{36}+\frac{7}{18}Y+\frac{1}{18}\pi^2-\frac{1}{3}XY-\frac{11}{18}X\right)+\frac{8}{9}\frac{u^2}{t^2}Y+\right.\\
&\quad\left.+\frac{u}{t}\left(\frac{2}{3}Y+\frac{8}{9}\right)\right)
\end{aligned} \tag{A.26}
$$

$$
F^{(2),[2]}_{s+--+} = \left(-\frac{25}{81}+\frac{1}{9}\pi^2\right)-\frac{10}{27}i\pi. \tag{A.27}
$$

A.3 $q(p_2,+)+\bar{Q}(p_1,-)\to q(p_3,+)+\bar{Q}(p_4,-)$

$$
\begin{aligned}
A^{(2),[1]}_{+-+-} = {}& \frac{t^2}{u^2}\left(-\frac{3}{2}\mathrm{Li}_4(x)+X\mathrm{Li}_3(x)-\frac{1}{4}X^2\mathrm{Li}_2(x)-X\zeta_3-\frac{37}{36}X\pi^2-\frac{29}{72}X^3+\frac{1}{60}\pi^4+\right.\\
&\quad\left.+\frac{16}{9}X^2-\frac{1}{4}X^2\pi^2+\frac{1}{24}X^4\right)+\\
&+\frac{t}{u}\left(-3\mathrm{Li}_4(x)+2X\mathrm{Li}_3(x)-\frac{1}{2}\mathrm{Li}_3(x)+\frac{1}{2}X\mathrm{Li}_2(x)-\frac{1}{2}X^2\mathrm{Li}_2(x)-\zeta_3\right.\\
&\quad-2X\zeta_3+\frac{1}{12}X^4-\frac{37}{36}\pi^2-\frac{5}{9}X^3+\frac{32}{9}X+\frac{1}{30}\pi^4-\frac{49}{18}X\pi^2+\frac{1}{4}X^2Y-\\
&\quad\left.-\frac{1}{2}X^2\pi^2+\frac{14}{9}X^2\right)\\
&+\left(-\mathrm{Li}_4(x)+X\mathrm{Li}_3(x)-\frac{11}{6}\mathrm{Li}_3(x)+\frac{11}{6}X\mathrm{Li}_2(x)+\frac{1}{3}\pi^2\mathrm{Li}_2(x)-\frac{1}{2}X^2\mathrm{Li}_2(x)+\right.\\
&\quad+\frac{1}{2}X\zeta_3-\frac{7}{2}Y\zeta_3+\frac{197}{72}\zeta_3-\frac{7}{9}X^3-\frac{23}{9}X-\frac{1}{96}\pi^4-\frac{49}{18}\pi^2+\frac{1}{8}X^4-\\
&\quad-\frac{373}{144}X\pi^2+\frac{1}{3}XY\pi^2+\frac{11}{12}X^2Y-\frac{7}{12}X^2\pi^2-\frac{409}{432}Y+\frac{23213}{5184}+
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{31}{12}X^2-\frac{1}{6}X^3Y+\frac{11}{48}Y\pi^2\bigg)+\\
&+i\pi\Bigg(\frac{t}{u}\bigg(2\mathrm{Li}_3(x)-X\mathrm{Li}_2(x)+\frac{1}{2}\mathrm{Li}_2(x)-2\zeta_3+\frac{1}{2}XY+\frac{32}{9}-\frac{1}{3}X\pi^2+\\
&\qquad+\frac{1}{3}X^3+\frac{1}{6}X^2-\frac{1}{6}\pi^2+\frac{89}{18}X\bigg)+\\
&\quad+\bigg(\mathrm{Li}_3(x)+\frac{11}{6}\mathrm{Li}_2(x)-X\mathrm{Li}_2(x)-3\zeta_3-\frac{1}{2}X^2Y-\frac{1}{6}X\pi^2-\frac{1}{2}X^2-\\
&\qquad-\frac{1}{4}\pi^2+\frac{11}{6}XY+\frac{1}{2}X^3+\frac{95}{48}+\frac{71}{18}X\bigg)+\\
&\quad+\frac{t^2}{u^2}\bigg(\mathrm{Li}_3(x)-\frac{1}{2}X\mathrm{Li}_2(x)-\zeta_3-\frac{7}{24}X^2-\frac{1}{6}X\pi^2+\frac{32}{9}X+\frac{1}{6}X^3\bigg)\Bigg)
\end{aligned}
\tag{A.28}
$$

$$
\begin{aligned}
B^{(2),[1]}_{+-+-}=\;&\frac{t}{u}\bigg(-5\mathrm{Li}_4(x)-10\mathrm{Li}_4(y)+10\mathrm{Li}_4(z)-8Y\mathrm{Li}_3(x)-\frac{13}{2}\mathrm{Li}_3(x)+8X\mathrm{Li}_3(x)-\\
&-6\mathrm{Li}_3(y)+4X\mathrm{Li}_3(y)+2XY\mathrm{Li}_2(x)+\frac{13}{2}X\mathrm{Li}_2(x)-\frac{1}{2}X^2\mathrm{Li}_2(x)-\\
&-\frac{7}{3}\pi^2\mathrm{Li}_2(x)+6Y\mathrm{Li}_2(y)+\zeta_3-8X\zeta_3+8Y\zeta_3+3XY^2+2X^2Y^2+\\
&+\frac{5}{6}Y^2\pi^2-\frac{5}{3}XY\pi^2-\frac{5}{3}XY^3+\frac{7}{6}Y\pi^2+\frac{35}{18}X^3-\frac{10}{9}X^2+\frac{343}{36}X\pi^2-\\
&-\frac{37}{9}X+\frac{47}{36}\pi^2+\frac{23}{36}\pi^4-\frac{3}{4}X^2Y+\frac{5}{12}Y^4+\frac{1}{4}XY-\frac{1}{3}X^3Y+\frac{37}{12}X^2\pi^2\bigg)+\\
&+\frac{t^2}{u^2}\bigg(-\frac{11}{2}\mathrm{Li}_4(x)-8\mathrm{Li}_4(y)+8\mathrm{Li}_4(z)+7X\mathrm{Li}_3(x)-\mathrm{Li}_3(x)-7Y\mathrm{Li}_3(x)+\\
&\quad+2X\mathrm{Li}_3(y)++XY\mathrm{Li}_2(x)-\frac{1}{4}X^2\mathrm{Li}_2(x)+X\mathrm{Li}_2(x)-\frac{5}{3}\pi^2\mathrm{Li}_2(x)+\\
&\quad+7Y\zeta_3-7X\zeta_3+\zeta_3+\frac{1}{8}X^2Y+\frac{49}{24}X^2\pi^2-\frac{1}{6}X^3Y+\frac{107}{36}X\pi^2+\\
&\quad+\frac{1}{3}Y^4-\frac{14}{9}X^2+\frac{1}{8}X^4-\frac{5}{6}XY\pi^2+\frac{23}{36}X^3-\frac{4}{3}XY^3+\frac{2}{3}Y^2\pi^2+\\
&\quad+X^2Y^2+\frac{89}{180}\pi^4\bigg)+\frac{t}{s}\bigg(5XY-\frac{5}{2}X^2-\frac{5}{2}Y^2-\frac{5}{2}\pi^2\bigg)+\\
&+\bigg(3\mathrm{Li}_4(x)-3X\mathrm{Li}_3(x)+\frac{22}{3}\mathrm{Li}_3(x)+\frac{11}{3}\mathrm{Li}_3(y)-\pi^2\mathrm{Li}_2(x)-\frac{22}{3}X\mathrm{Li}_2(x)+\\
&+\frac{3}{2}X^2\mathrm{Li}_2(x)-\frac{11}{3}Y\mathrm{Li}_2(y)-\frac{1}{2}Y\zeta_3-\frac{707}{72}\zeta_3-\frac{1}{2}XY^3+\frac{7}{6}XY^2-\\
&-\frac{7}{6}XY\pi^2+\frac{599}{72}Y^2+\frac{25}{6}X\pi^2+\frac{5}{12}Y^2\pi^2+\frac{1}{8}Y^4+2X^2\pi^2-\frac{8}{3}X^2Y-\\
&-\frac{319}{144}Y\pi^2+2X^2+X^2Y^2-\frac{26}{3}X-\frac{275}{54}Y+\frac{163}{16}\pi^2-\frac{77}{1440}\pi^4+\\
&+\frac{30659}{1296}-\frac{71}{36}Y^3-\frac{71}{9}XY-\frac{1}{2}X^3Y\bigg)+\\
&+i\pi\Bigg(\bigg(-3\mathrm{Li}_3(x)-Y\mathrm{Li}_2(x)+3X\mathrm{Li}_2(x)-\frac{11}{3}\mathrm{Li}_2(x)-Y\mathrm{Li}_2(y)-\frac{1}{2}\zeta_3+\\
&\qquad+\frac{3}{4}Y^2-\frac{9}{4}Y+\frac{1}{2}X^2Y-X^3+\frac{5}{6}X\pi^2+\frac{5}{6}Y\pi^2-\frac{35}{9}X+\frac{49}{54}+\frac{83}{48}\pi^2-\\
&\qquad-\frac{1}{2}XY^2-\frac{20}{3}XY+X^2\bigg)+
\end{aligned}
$$

JHEP04(2004)021

$$
\begin{aligned}
&+\frac{t}{u}\bigg(4\mathrm{Li}_3(y)+\frac{1}{2}\mathrm{Li}_2(x)-Y\mathrm{Li}_2(x)+X\mathrm{Li}_2(x)-3Y\mathrm{Li}_2(y)-\frac{203}{36}X+\frac{1}{4}\pi^2+\\
&\qquad+\frac{1}{4}Y+\frac{1}{6}Y\pi^2-\frac{37}{9}-\frac{4}{3}X^3-XY^2-\frac{3}{2}XY-\frac{11}{6}X^2+\frac{1}{2}X\pi^2\bigg)+\\
&+\frac{t^2}{u^2}\bigg(2\mathrm{Li}_3(y)-\frac{1}{2}Y\mathrm{Li}_2(x)+\mathrm{Li}_2(x)+\frac{1}{2}X\mathrm{Li}_2(x)-\frac{3}{2}Y\mathrm{Li}_2(y)-\frac{7}{24}X^2+\\
&\qquad+\frac{1}{4}XY-\frac{1}{6}\pi^2+\frac{1}{4}X\pi^2-\frac{28}{9}X-\frac{2}{3}X^3+\frac{1}{12}Y\pi^2-\frac{1}{2}XY^2\bigg)\bigg)
\end{aligned}
\tag{A.29}
$$

$$
\begin{aligned}
C^{(2),[1]}_{+-+-} = &\bigg(-4\mathrm{Li}_4(x)-8\mathrm{Li}_4(z)-4X\mathrm{Li}_3(x)+8Y\mathrm{Li}_3(x)+8Y\mathrm{Li}_3(y)-8X\mathrm{Li}_3(y)+\\
&+\frac{8}{3}\pi^2\mathrm{Li}_2(x)-10XY\mathrm{Li}_2(x)+2X^2\mathrm{Li}_2(x)-4Y^2\mathrm{Li}_2(y)-2XY\mathrm{Li}_2(y)+\\
&+8X\zeta_3-5Y\zeta_3-\frac{15}{4}\zeta_3-\frac{7}{24}Y^4+\frac{59}{8}Y^2+\frac{9}{2}XY^2+\frac{17}{6}XY\pi^2+\frac{2}{3}X^3Y-\\
&-\frac{3}{2}X^2\pi^2-\frac{9}{2}X\pi^2-\frac{5}{2}Y\pi^2-\frac{13}{2}X^2Y^2-\frac{1}{6}XY^3-\frac{19}{12}Y^2\pi^2-\frac{189}{16}Y+\\
&+\frac{59}{48}\pi^2+\frac{31}{90}\pi^4+\frac{511}{64}-\frac{9}{4}Y^3-8XY\bigg)+\\
&+\frac{t^2}{u^2}\bigg(6\mathrm{Li}_4(x)+6\mathrm{Li}_4(y)-6\mathrm{Li}_4(z)-8X\mathrm{Li}_3(x)+3\mathrm{Li}_3(x)+3Y\mathrm{Li}_3(x)-6X\mathrm{Li}_3(y)-\\
&\qquad-3XY\mathrm{Li}_2(x)+2X^2\mathrm{Li}_2(x)-3X\mathrm{Li}_2(x)+2\pi^2\mathrm{Li}_2(x)-3Y\zeta_3+8X\zeta_3-\\
&\qquad-3\zeta_3-\frac{3}{8}X^2Y-\frac{35}{24}X^2\pi^2+\frac{1}{2}X^3Y-\frac{29}{12}X\pi^2-\frac{1}{4}Y^4-\frac{9}{2}X^2+\frac{1}{12}X^4+\\
&\qquad+\frac{5}{2}XY\pi^2+\frac{1}{3}X^3+XY^3-\frac{1}{2}Y^2\pi^2-3X^2Y^2-\frac{31}{60}\pi^4\bigg)+\\
&+\frac{t}{u}\bigg(12\mathrm{Li}_4(x)+12\mathrm{Li}_4(y)-12\mathrm{Li}_4(z)+4\mathrm{Li}_3(x)+6Y\mathrm{Li}_3(x)-16X\mathrm{Li}_3(x)-\\
&\qquad-12X\mathrm{Li}_3(y)+4\pi^2\mathrm{Li}_2(x)-6XY\mathrm{Li}_2(x)-4X\mathrm{Li}_2(x)+4X^2\mathrm{Li}_2(x)+\\
&\qquad+16X\zeta_3+2\zeta_3-6Y\zeta_3+X^2Y-Y^2\pi^2-\frac{71}{12}X\pi^2-6X^2Y^2-\frac{35}{12}X^2\pi^2-\\
&\qquad-\frac{1}{2}Y\pi^2+\frac{1}{3}X^3-\frac{9}{2}X^2+5XY\pi^2+2XY^3-6X-\frac{5}{12}\pi^2+\frac{1}{6}X^4-\\
&\qquad-\frac{1}{2}Y^4-\frac{3}{4}XY+X^3Y-\frac{31}{30}\pi^4\bigg)+\\
&+\frac{t}{s}\bigg(3XY-\frac{3}{2}X^2-\frac{3}{2}Y^2-\frac{3}{2}\pi^2\bigg)+\\
&+i\pi\Bigg(\frac{t^2}{u^2}\bigg(-5\mathrm{Li}_3(x)-6\mathrm{Li}_3(y)-3\mathrm{Li}_2(x)+X\mathrm{Li}_2(x)+\frac{3}{2}Y\mathrm{Li}_2(x)+\\
&\qquad+\frac{9}{2}Y\mathrm{Li}_2(y)+5\zeta_3+\frac{5}{6}X^3+\frac{1}{2}\pi^2+\frac{17}{8}X^2-9X+\frac{1}{4}X\pi^2-\\
&\qquad-\frac{1}{4}Y\pi^2+\frac{3}{2}XY^2-\frac{3}{4}XY\bigg)+\\
&+\frac{t}{u}\bigg(-10\mathrm{Li}_3(x)-12\mathrm{Li}_3(y)-4\mathrm{Li}_2(x)+2X\mathrm{Li}_2(x)+3Y\mathrm{Li}_2(x)+9Y\mathrm{Li}_2(y)+\\
&\qquad+10\zeta_3+\frac{1}{2}X\pi^2+3XY^2+2XY+4X^2-\frac{39}{4}X-6+\frac{5}{3}X^3-\\
&\qquad-\frac{1}{2}Y\pi^2+\frac{1}{4}\pi^2-\frac{3}{4}Y\bigg)+
\end{aligned}
$$

$$+\Big(4\mathrm{Li}_3(x) - 4X\mathrm{Li}_2(x) + 3Y\mathrm{Li}_2(x) + 3Y\mathrm{Li}_2(y) + 3\zeta_3 - \frac{5}{2}\pi^2 + 9XY + \frac{27}{4}Y + X^2Y - \frac{189}{16} - 8X - \frac{3}{2}X\pi^2 - \frac{9}{4}Y^2 - \frac{5}{2}Y\pi^2 + \frac{3}{2}XY^2\Big)\Big) \tag{A.30}$$

$$\begin{aligned}
D^{(2),[1]}_{+-+-} = {} & \frac{t^2}{u^2}\Big(\frac{1}{9}X\pi^2 + \frac{1}{9}X^3 - \frac{4}{9}X^2\Big) + \frac{t}{u}\Big(\frac{2}{9}X^3 - \frac{7}{18}X^2 - \frac{8}{9}X + \frac{2}{9}X\pi^2 + \frac{1}{9}\pi^2\Big) + \\
& + \Big(\frac{1}{3}\mathrm{Li}_3(x) - \frac{1}{3}X\mathrm{Li}_2(x) - \frac{49}{36}\zeta_3 - \frac{1}{6}X^2Y - \frac{1}{24}Y\pi^2 + \frac{265}{108}X + \frac{11}{72}X\pi^2 + \\
& \quad + \frac{5}{18}X^3 + \frac{25}{72}\pi^2 - \frac{37}{36}X^2 + \frac{25}{108}Y - \frac{455}{108}\Big) + \\
& + i\pi\Big(\Big(-\frac{1}{3}\mathrm{Li}_2(x) + \frac{1}{2}X^2 - \frac{1}{3}XY - \frac{11}{18}X - \frac{11}{12}\Big) + \\
& \quad + \frac{t}{u}\Big(-\frac{10}{9}X - \frac{8}{9} + \frac{1}{3}X^2\Big) + \frac{t^2}{u^2}\Big(\frac{1}{6}X^2 - \frac{8}{9}X\Big)\Big)
\end{aligned} \tag{A.31}$$

$$\begin{aligned}
E^{(2),[1]}_{+-+-} = {} & \frac{t^2}{u^2}\Big(\frac{8}{9}X^2 - \frac{2}{9}X^3 - \frac{2}{9}X\pi^2\Big) + \\
& + \frac{t}{u}\Big(-\frac{4}{9}X\pi^2 + \frac{16}{9}X - \frac{4}{9}X^3 + \frac{7}{9}X^2 - \frac{2}{9}\pi^2\Big) + \\
& + \Big(-\frac{4}{3}\mathrm{Li}_3(x) - \frac{2}{3}\mathrm{Li}_3(y) + \frac{4}{3}X\mathrm{Li}_2(x) + \frac{2}{3}Y\mathrm{Li}_2(y) - \frac{11}{36}\zeta_3 + \frac{223}{108}Y - \\
& \quad - \frac{1}{3}X^2Y + \frac{2}{9}Y^3 + \frac{11}{9}XY + \frac{29}{72}Y\pi^2 - \frac{41}{24}\pi^2 + \frac{1}{3}XY^2 - \frac{29}{18}Y^2 + \\
& \quad + \frac{29}{12}X + \frac{1}{3}X\pi^2 - \frac{685}{162}\Big) + \\
& + i\pi\Big(\frac{t}{u}\Big(-\frac{2}{3}X^2 + \frac{16}{9} + \frac{20}{9}X\Big) + \\
& \qquad + \Big(\frac{2}{3}\mathrm{Li}_2(x) - \frac{1}{24}\pi^2 + \frac{223}{108} + \frac{2}{3}XY - X^2 + \frac{11}{9}X\Big) \\
& \qquad + \frac{t^2}{u^2}\Big(-\frac{1}{3}X^2 + \frac{16}{9}X\Big)\Big)
\end{aligned} \tag{A.32}$$

$$F^{(2),[1]}_{+-+-} = \Big(\frac{25}{81} - \frac{10}{27}X + \frac{1}{9}X^2\Big) \tag{A.33}$$

$$\begin{aligned}
A^{(2),[2]}_{+-+-} = {} & \frac{t^2}{u^2}\Big(5\mathrm{Li}_4(x) + 8\mathrm{Li}_4(y) - 8\mathrm{Li}_4(z) - 7X\mathrm{Li}_3(x) - \frac{1}{2}\mathrm{Li}_3(x) + 7Y\mathrm{Li}_3(x) - 2X\mathrm{Li}_3(y) - \\
& \quad - XY\mathrm{Li}_2(x) + \frac{1}{2}X^2\mathrm{Li}_2(x) + \frac{1}{2}X\mathrm{Li}_2(x) + \frac{4}{3}\pi^2\mathrm{Li}_2(x) - 7Y\zeta_3 + 7X\zeta_3 + \\
& \quad + \frac{1}{2}\zeta_3 - \frac{1}{8}X^2Y - \frac{29}{24}X^2\pi^2 + \frac{1}{6}X^3Y + \frac{1}{12}X\pi^2 - \frac{1}{3}Y^4 - \frac{5}{4}X^2 - \frac{5}{24}X^4 + \\
& \quad + \frac{5}{6}XY\pi^2 + \frac{7}{24}X^3 + \frac{4}{3}XY^3 - \frac{2}{3}Y^2\pi^2 - X^2Y^2 - \frac{13}{30}\pi^4\Big) + \\
& + \frac{t}{u}\Big(4\mathrm{Li}_4(x) + 10\mathrm{Li}_4(y) - 10\mathrm{Li}_4(z) + 8Y\mathrm{Li}_3(x) + 3\mathrm{Li}_3(x) - 8X\mathrm{Li}_3(x) + 6\mathrm{Li}_3(y) - \\
& \quad - 4X\mathrm{Li}_3(y) - 2XY\mathrm{Li}_2(x) - 3X\mathrm{Li}_2(x) + X^2\mathrm{Li}_2(x) + \frac{5}{3}\pi^2\mathrm{Li}_2(x) - \\
& \quad - 6Y\mathrm{Li}_2(y) + 2\zeta_3 + 8X\zeta_3 - 8Y\zeta_3 - 3XY^2 - 2X^2Y^2 - \frac{5}{6}Y^2\pi^2 +
\end{aligned}$$

$$
\begin{aligned}
&+\frac{5}{3}XY\pi^2-\frac{19}{12}X\pi^2-\frac{5}{6}Y\pi^2-\frac{1}{2}X^3+\frac{1}{4}X^2+\frac{1}{2}X^2Y+\frac{5}{3}XY^3+\\
&+\frac{17}{12}\pi^2-\frac{31}{60}\pi^4-\frac{1}{6}X^4-\frac{5}{12}Y^4-\frac{7}{4}XY+\frac{1}{3}X^3Y-\frac{17}{12}X^2\pi^2\Big)+\\
&+\frac{t}{s}\left(\frac{3}{2}Y^2+\frac{3}{2}X^2-3XY+\frac{3}{2}\pi^2\right)+\\
&+\Big(-2\mathrm{Li}_4(x)+4\mathrm{Li}_4(y)+\mathrm{Li}_4(z)+X\mathrm{Li}_3(x)-\frac{29}{6}\mathrm{Li}_3(x)-\frac{10}{3}\mathrm{Li}_3(y)-\\
&-3Y\mathrm{Li}_3(y)-2XY\mathrm{Li}_2(x)+\frac{29}{6}X\mathrm{Li}_2(x)+\frac{1}{2}X^2\mathrm{Li}_2(x)-\frac{1}{6}\pi^2\mathrm{Li}_2(x)-\\
&-XY\mathrm{Li}_2(y)+\frac{1}{2}Y^2\mathrm{Li}_2(y)+\frac{10}{3}Y\mathrm{Li}_2(y)-\frac{1}{2}X\zeta_3+4Y\zeta_3+\frac{43}{72}\zeta_3+\\
&+\frac{289}{144}X\pi^2-\frac{277}{72}Y^2-\frac{1}{6}XY\pi^2-\frac{7}{24}X^2\pi^2+\frac{11}{18}Y^3+\frac{7}{12}X^3Y+\\
&+\frac{11}{12}XY^2-\frac{9}{8}X^2Y^2+\frac{203}{72}XY-\frac{7}{12}XY^3+\frac{5}{8}Y^2\pi^2+\frac{23}{9}X+\\
&+\frac{1673}{432}Y-2\pi^2-\frac{109}{1440}\pi^4-\frac{23213}{5184}+\frac{5}{12}X^2Y+\frac{55}{72}Y\pi^2+\\
&+\frac{7}{9}X^3-\frac{31}{12}X^2-\frac{1}{8}X^4\Big)+\\
&+i\pi\Bigg(\frac{t^2}{u^2}\Big(-2\mathrm{Li}_3(y)+\frac{1}{2}Y\mathrm{Li}_2(x)+\frac{1}{2}\mathrm{Li}_2(x)+\frac{3}{2}Y\mathrm{Li}_2(y)-\frac{5}{2}X+\frac{1}{2}X^2+\\
&\qquad+\frac{1}{12}X\pi^2+\frac{1}{6}X^3-\frac{1}{12}\pi^2+\frac{1}{2}XY^2-\frac{1}{12}Y\pi^2-\frac{1}{4}XY\Big)+\\
&\quad+\frac{t}{u}\Big(-4\mathrm{Li}_3(y)+3\mathrm{Li}_2(x)+Y\mathrm{Li}_2(x)+3Y\mathrm{Li}_2(y)+\frac{1}{3}X^3+XY+\frac{1}{6}X\pi^2-\\
&\qquad-\frac{5}{4}X-\frac{7}{4}Y+\frac{1}{2}X^2+XY^2-\frac{1}{6}Y\pi^2-\frac{5}{12}\pi^2\Big)+\\
&\quad+\Big(\mathrm{Li}_3(x)-3\mathrm{Li}_3(y)+\frac{3}{2}\mathrm{Li}_2(x)+Y\mathrm{Li}_2(x)+3Y\mathrm{Li}_2(y)+\frac{7}{2}\zeta_3-\frac{5}{12}X^3-\\
&\qquad-\frac{59}{48}\pi^2-\frac{5}{12}Y^3+\frac{409}{432}+\frac{5}{8}Y+\frac{1}{4}Y\pi^2-\frac{5}{12}X\pi^2-\frac{3}{2}X^2-\frac{3}{4}Y^2+\\
&\qquad+2X^2Y+3XY-\frac{9}{8}X+\frac{1}{2}XY^2\Big)\Bigg)
\end{aligned}
\tag{A.34}
$$

$$
\begin{aligned}
B^{(2),[2]}_{+-+-}=\;&\frac{t^2}{u^2}\Big(-\frac{3}{2}\mathrm{Li}_4(x)-4\mathrm{Li}_4(y)+4\mathrm{Li}_4(z)+4X\mathrm{Li}_3(x)-\frac{1}{2}\mathrm{Li}_3(x)-2Y\mathrm{Li}_3(x)+\\
&+4X\mathrm{Li}_3(y)-\frac{5}{4}X^2\mathrm{Li}_2(x)+\frac{1}{2}X\mathrm{Li}_2(x)+2XY\mathrm{Li}_2(x)-\pi^2\mathrm{Li}_2(x)-\\
&-4X\zeta_3+2Y\zeta_3+\frac{1}{2}\zeta_3+\frac{1}{3}X^2\pi^2+\frac{1}{6}Y^4-\frac{1}{3}X^3Y-\frac{19}{36}X\pi^2-\frac{25}{36}X^3+\\
&+\frac{127}{36}X^2+\frac{1}{4}X^2Y-\frac{5}{3}XY\pi^2+2X^2Y^2-\frac{2}{3}XY^3+\frac{1}{3}Y^2\pi^2+\frac{47}{180}\pi^4\Big)+\\
&+\frac{t}{u}\Big(-3\mathrm{Li}_4(x)-8\mathrm{Li}_4(y)+8\mathrm{Li}_4(z)-4Y\mathrm{Li}_3(x)+8X\mathrm{Li}_3(x)+\frac{3}{2}\mathrm{Li}_3(x)+\\
&+8X\mathrm{Li}_3(y)-2\pi^2\mathrm{Li}_2(x)-\frac{5}{2}X^2\mathrm{Li}_2(x)+4XY\mathrm{Li}_2(x)-\frac{3}{2}X\mathrm{Li}_2(x)-\\
&-8X\zeta_3+4Y\zeta_3-3\zeta_3-\frac{25}{18}X\pi^2-\frac{3}{4}X^2Y+\frac{2}{3}X^2\pi^2+\frac{2}{3}Y^2\pi^2-
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \frac{2}{3}X^3Y - \frac{4}{3}XY^3 - \frac{19}{18}X^3 + \frac{65}{36}X^2 - \frac{10}{3}XY\pi^2 - \frac{55}{36}\pi^2 + \frac{47}{90}\pi^4 + \\
&\quad + 4X^2Y^2 + \frac{32}{9}X + \frac{1}{3}Y^4 + 2XY\Big) + \\
&+\frac{t}{s}\Big(2X^2 - 4XY + 2\pi^2 + 2Y^2\Big) + \\
&+\Big(3\mathrm{Li}_4(x) - 4\mathrm{Li}_4(y) + 4\mathrm{Li}_4(z) + 3X\mathrm{Li}_3(x) - \frac{2}{3}\mathrm{Li}_3(x) - 5Y\mathrm{Li}_3(x) - \frac{1}{3}\mathrm{Li}_3(y) + \\
&\quad + 5X\mathrm{Li}_3(y) - 2Y\mathrm{Li}_3(y) + 8XY\mathrm{Li}_2(x) - \pi^2\mathrm{Li}_2(x) - \frac{5}{2}X^2\mathrm{Li}_2(x) + \\
&\quad + \frac{2}{3}X\mathrm{Li}_2(x) + 2XY\mathrm{Li}_2(y) + 2Y^2\mathrm{Li}_2(y) + \frac{1}{3}Y\mathrm{Li}_2(y) - 5X\zeta_3 + \frac{467}{72}\zeta_3 + \\
&\quad + 5Y\zeta_3 - \frac{1}{8}X^2\pi^2 - \frac{5}{6}XY\pi^2 - \frac{7}{12}X\pi^2 - \frac{157}{18}Y^2 + \frac{4}{3}X^2Y + \frac{653}{72}XY + \\
&\quad + \frac{1}{24}Y^4 + \frac{5}{4}XY^3 + \frac{103}{36}Y^3 + \frac{31}{8}X^2Y^2 + \frac{26}{3}X + \frac{49}{6}Y - \frac{1}{24}Y^2\pi^2 - \\
&\quad - \frac{53}{480}\pi^4 - \frac{30659}{1296} - \frac{1}{4}X^3Y - \frac{751}{144}\pi^2 - \frac{61}{12}XY^2 + \frac{49}{18}Y\pi^2 - 2X^2\Big) + \\
&+i\pi\Big(\frac{t^2}{u^2}\Big(2\mathrm{Li}_3(x) + 4\mathrm{Li}_3(y) + \frac{1}{2}\mathrm{Li}_2(x) - Y\mathrm{Li}_2(x) - \frac{1}{2}X\mathrm{Li}_2(x) - 3Y\mathrm{Li}_2(y) - \\
&\qquad - 2\zeta_3 - XY^2 - \frac{1}{6}X^3 - \frac{7}{6}X^2 - \frac{1}{3}X\pi^2 + \frac{1}{2}XY - \frac{1}{12}\pi^2 + \\
&\qquad + \frac{1}{6}Y\pi^2 + \frac{127}{18}X\Big) + \\
&+\frac{t}{u}\Big(4\mathrm{Li}_3(x) + 8\mathrm{Li}_3(y) - \frac{3}{2}\mathrm{Li}_2(x) - 2Y\mathrm{Li}_2(x) - X\mathrm{Li}_2(x) - 6Y\mathrm{Li}_2(y) - \\
&\quad - 4\zeta_3 + \frac{32}{9} - 2XY^2 - \frac{2}{3}X\pi^2 + \frac{1}{3}Y\pi^2 - \frac{3}{2}XY + \frac{67}{9}X + \frac{1}{6}\pi^2 - \\
&\quad - \frac{4}{3}X^2 - \frac{1}{3}X^3 + 2Y\Big) + \\
&+\Big(-2\mathrm{Li}_3(x) + 3\mathrm{Li}_3(y) + X\mathrm{Li}_2(x) - 2Y\mathrm{Li}_2(x) + \frac{1}{3}\mathrm{Li}_2(x) - 4Y\mathrm{Li}_2(y) - \\
&\quad - \frac{25}{6}XY + \frac{365}{72}X + \frac{11}{12}X^3 - XY^2 + \frac{7}{12}Y\pi^2 + \frac{3}{2}Y^2 + \frac{5}{12}Y^3 + \frac{5}{4}\pi^2 + \\
&\quad + \frac{13}{6} - \frac{5}{2}X^2Y + X^2 - \frac{23}{8}Y + \frac{3}{4}X\pi^2\Big)\Big)
\end{aligned}
\tag{A.35}
$$

$$
\begin{aligned}
C^{(2),[2]}_{+-+-} = \frac{t^2}{u^2}\Big(&-\frac{5}{2}\mathrm{Li}_4(x) - 2\mathrm{Li}_4(y) + 2\mathrm{Li}_4(z) + 3X\mathrm{Li}_3(x) - \mathrm{Li}_3(x) - Y\mathrm{Li}_3(x) + \\
&+ 2X\mathrm{Li}_3(y) + XY\mathrm{Li}_2(x) - \frac{3}{4}X^2\mathrm{Li}_2(x) + X\mathrm{Li}_2(x) - \frac{2}{3}\pi^2\mathrm{Li}_2(x) + Y\zeta_3 - \\
&- 3X\zeta_3 + \zeta_3 + \frac{1}{8}X^2Y + \frac{13}{24}X^2\pi^2 - \frac{1}{6}X^3Y + \frac{11}{12}X\pi^2 + \frac{1}{12}Y^4 + 2X^2 - \\
&- \frac{1}{24}X^4 - \frac{5}{6}XY\pi^2 - \frac{1}{6}X^3 - \frac{1}{3}XY^3 + \frac{1}{6}Y^2\pi^2 + X^2Y^2 + \frac{8}{45}\pi^4\Big) + \\
+\Big(&\mathrm{Li}_4(x) + 3\mathrm{Li}_4(z) - 3Y\mathrm{Li}_3(x) + 2X\mathrm{Li}_3(x) + 3X\mathrm{Li}_3(y) - 3Y\mathrm{Li}_3(y) + \\
&+ 4XY\mathrm{Li}_2(x) - \frac{5}{6}\pi^2\mathrm{Li}_2(x) - X^2\mathrm{Li}_2(x) + \frac{3}{2}Y^2\mathrm{Li}_2(y) + XY\mathrm{Li}_2(y) +
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{15}{4}\zeta_3-3X\zeta_3+\frac{1}{2}X^2\pi^2+\frac{1}{8}Y^4+\frac{7}{12}Y^2\pi^2+\frac{3}{2}X\pi^2-\frac{1}{3}X^3Y-\\
&-XY\pi^2+Y\pi^2+4XY-\frac{3}{2}XY^2+\frac{11}{4}X^2Y^2+\frac{93}{16}Y-\frac{71}{48}\pi^2-\\
&-\frac{17}{180}\pi^4-\frac{511}{64}-\frac{25}{8}Y^2+\frac{3}{4}Y^3\Bigg)+\\
&+\frac{t}{u}\Bigg(-5\mathrm{Li}_4(x)-4\mathrm{Li}_4(y)+4\mathrm{Li}_4(z)-\frac{3}{2}\mathrm{Li}_3(x)-2Y\mathrm{Li}_3(x)+6X\mathrm{Li}_3(x)+\\
&\quad+4X\mathrm{Li}_3(y)-\frac{4}{3}\pi^2\mathrm{Li}_2(x)+2XY\mathrm{Li}_2(x)+\frac{3}{2}X\mathrm{Li}_2(x)-\frac{3}{2}X^2\mathrm{Li}_2(x)-\\
&\quad-6X\zeta_3-\zeta_3+2Y\zeta_3-\frac{1}{4}X^2Y+\frac{1}{3}Y^2\pi^2+\frac{25}{12}X\pi^2+2X^2Y^2+\\
&\quad+\frac{13}{12}X^2\pi^2+\frac{1}{6}Y\pi^2-\frac{1}{6}X^3+2X^2-\frac{5}{3}XY\pi^2-\frac{2}{3}XY^3+3X+\\
&\quad+\frac{1}{4}\pi^2-\frac{1}{12}X^4+\frac{1}{6}Y^4+\frac{1}{4}XY-\frac{1}{3}X^3Y+\frac{16}{45}\pi^4\Bigg)+\\
&+\frac{t}{s}\Bigg(-XY+\frac{1}{2}\pi^2+\frac{1}{2}Y^2+\frac{1}{2}X^2\Bigg)+\\
&+i\pi\Bigg(\frac{t^2}{u^2}\Bigg(2\mathrm{Li}_3(x)+2\mathrm{Li}_3(y)-\frac{1}{2}X\mathrm{Li}_2(x)-\frac{1}{2}Y\mathrm{Li}_2(x)+\mathrm{Li}_2(x)-\frac{3}{2}Y\mathrm{Li}_2(y)-\\
&\qquad-2\zeta_3-\frac{1}{3}X^3+\frac{1}{12}Y\pi^2-\frac{7}{8}X^2-\frac{1}{2}XY^2-\frac{1}{6}\pi^2+\\
&\qquad+\frac{1}{4}XY+4X-\frac{1}{12}X\pi^2\Bigg)+\\
&\quad+\frac{t}{u}\Bigg(4\mathrm{Li}_3(x)+4\mathrm{Li}_3(y)-X\mathrm{Li}_2(x)-Y\mathrm{Li}_2(x)+\frac{3}{2}\mathrm{Li}_2(x)-3Y\mathrm{Li}_2(y)-4\zeta_3+\\
&\qquad+3-\frac{1}{2}XY+\frac{1}{4}Y-\frac{3}{2}X^2-\frac{1}{12}\pi^2-\frac{2}{3}X^3-\frac{1}{6}X\pi^2+\\
&\qquad+\frac{17}{4}X+\frac{1}{6}Y\pi^2-XY^2\Bigg)+\\
&\quad+\Bigg(-\mathrm{Li}_3(x)+X\mathrm{Li}_2(x)-Y\mathrm{Li}_2(x)-Y\mathrm{Li}_2(y)-3\zeta_3+4X+\pi^2-\frac{9}{4}Y+\\
&\qquad+\frac{1}{2}X\pi^2-\frac{1}{2}X^2Y+\frac{5}{6}Y\pi^2+\frac{93}{16}-\frac{1}{2}XY^2-3XY+\frac{3}{4}Y^2\Bigg)\Bigg)
\end{aligned}\tag{A.36}
$$

$$
\begin{aligned}
D^{(2),[2]}_{+-+-} &= \Bigg(\frac{1}{3}\mathrm{Li}_3(x)+\frac{1}{3}\mathrm{Li}_3(y)-\frac{1}{3}X\mathrm{Li}_2(x)-\frac{1}{3}Y\mathrm{Li}_2(y)+\frac{37}{36}\zeta_3+\frac{37}{36}X^2-\frac{149}{108}Y-\\
&\quad-\frac{23}{72}X\pi^2-\frac{11}{18}XY+\frac{29}{36}Y^2-\frac{1}{6}XY^2+\frac{11}{24}\pi^2-\frac{5}{18}X^3+\frac{1}{3}X^2Y-\\
&\quad-\frac{265}{108}X+\frac{455}{108}-\frac{1}{9}Y^3-\frac{5}{36}Y\pi^2\Bigg)+i\pi\Bigg(-\frac{25}{108}+\frac{1}{24}\pi^2\Bigg)
\end{aligned}\tag{A.37}
$$

$$
\begin{aligned}
E^{(2),[2]}_{+-+-} &= \frac{t^2}{u^2}\Bigg(\frac{1}{9}X\pi^2+\frac{1}{9}X^3-\frac{4}{9}X^2\Bigg)+\\
&\quad+\frac{t}{u}\Bigg(\frac{2}{9}X^3-\frac{7}{18}X^2-\frac{8}{9}X+\frac{2}{9}X\pi^2+\frac{1}{9}\pi^2\Bigg)+\\
&\quad+\Bigg(\frac{2}{3}\mathrm{Li}_3(x)+\frac{1}{3}\mathrm{Li}_3(y)-\frac{2}{3}X\mathrm{Li}_2(x)-\frac{1}{3}Y\mathrm{Li}_2(y)+\frac{23}{36}\zeta_3+\frac{1}{6}X^2Y+\frac{29}{36}Y^2-\\
&\quad-\frac{11}{18}XY-\frac{29}{12}X+\frac{685}{162}-\frac{1}{6}XY^2-\frac{2}{9}Y\pi^2-\frac{11}{12}Y-\frac{1}{9}Y^3+\frac{65}{72}\pi^2-\frac{1}{6}X\pi^2\Bigg)+
\end{aligned}
$$

$$
\begin{aligned}
&+i\pi\Bigg(\bigg(-\frac{1}{3}\mathrm{Li}_2(x)+\frac{1}{2}X^2-\frac{1}{3}XY-\frac{11}{18}X-\frac{11}{12}\bigg)+\\
&\qquad+\frac{t}{u}\bigg(-\frac{10}{9}X-\frac{8}{9}+\frac{1}{3}X^2\bigg)+\frac{t^2}{u^2}\bigg(\frac{1}{6}X^2-\frac{8}{9}X\bigg)\Bigg) \qquad (A.38)
\end{aligned}
$$

$$
F^{(2),[2]}_{+-+-} = \bigg(\frac{10}{27}X-\frac{1}{9}X^2-\frac{25}{81}\bigg). \qquad (A.39)
$$

A.4 $q(p_2,+)+\bar{Q}(p_1,+)\to q(p_3,+)+\bar{Q}(p_4,+)$

$$
\begin{aligned}
A^{(2),[1]}_{++++} =\; & -\frac{3}{2}\frac{t}{u}X^2+\frac{t^2}{s^2}\bigg(3\mathrm{Li}_4(x)+3\mathrm{Li}_4(y)-3\mathrm{Li}_4(z)-3X\mathrm{Li}_3(x)-3X\mathrm{Li}_3(y)+\\
&\qquad+\frac{1}{2}\pi^2\mathrm{Li}_2(x)-\frac{4}{15}\pi^4-\frac{1}{4}Y^2\pi^2+XY\pi^2-\frac{1}{2}X^3Y+\frac{1}{8}X^4-\\
&\qquad-\frac{1}{4}X^2\pi^2+\frac{1}{2}XY^3-\frac{3}{4}X^2Y^2-\frac{1}{8}Y^4\bigg)+\\
&+\frac{t}{s}\bigg(3\mathrm{Li}_4(x)+3\mathrm{Li}_4(y)-3\mathrm{Li}_4(z)+3\mathrm{Li}_3(x)-3X\mathrm{Li}_3(x)-3X\mathrm{Li}_3(y)+\\
&\quad+\frac{1}{2}\pi^2\mathrm{Li}_2(x)-3X\mathrm{Li}_2(x)-3\zeta_3-\frac{1}{2}\pi^2+XY\pi^2+\frac{1}{8}X^4-\frac{3}{2}X^2Y+\\
&\quad+\frac{1}{2}X^3-\frac{1}{8}Y^4-\frac{1}{4}X^2\pi^2+\frac{1}{2}XY^3-\frac{3}{4}X^2Y^2-\frac{1}{4}Y^2\pi^2-\\
&\quad-\frac{1}{2}X\pi^2-\frac{4}{15}\pi^4-\frac{1}{2}X^3Y\bigg)+\\
&+\bigg(-\mathrm{Li}_4(x)+X\mathrm{Li}_3(x)-\frac{11}{6}\mathrm{Li}_3(x)+\frac{11}{6}X\mathrm{Li}_2(x)+\frac{1}{3}\pi^2\mathrm{Li}_2(x)-\\
&\quad-\frac{1}{2}X^2\mathrm{Li}_2(x)+\frac{1}{2}X\zeta_3-\frac{7}{2}Y\zeta_3+\frac{197}{72}\zeta_3-\frac{7}{9}X^3-\frac{23}{9}X-\frac{1}{96}\pi^4-\\
&\quad-\frac{49}{18}\pi^2+\frac{1}{8}X^4-\frac{373}{144}X\pi^2++\frac{1}{3}XY\pi^2+\frac{11}{12}X^2Y-\frac{7}{12}X^2\pi^2-\\
&\quad-\frac{409}{432}Y+\frac{23213}{5184}+\frac{31}{12}X^2-\frac{1}{6}X^3Y+\frac{11}{48}Y\pi^2\bigg)+\\
&+i\pi\Bigg(-3\frac{t}{u}X+\frac{t}{s}\bigg(-3\mathrm{Li}_3(x)-3\mathrm{Li}_3(y)-3\mathrm{Li}_2(x)+\frac{3}{2}X^2+\frac{1}{2}X^3-\\
&\qquad\qquad-3XY-\frac{3}{2}X^2Y+\frac{1}{2}\pi^2+\frac{1}{2}X\pi^2\bigg)+\\
&\quad+\bigg(\mathrm{Li}_3(x)+\frac{11}{6}\mathrm{Li}_2(x)-X\mathrm{Li}_2(x)-3\zeta_3-\frac{1}{2}X^2Y-\frac{1}{6}X\pi^2-\frac{1}{2}X^2-\\
&\qquad-\frac{1}{4}\pi^2+\frac{11}{6}XY+\frac{1}{2}X^3+\frac{95}{48}+\frac{71}{18}X\bigg)+\\
&\quad+\frac{t^2}{s^2}\bigg(-3\mathrm{Li}_3(x)-3\mathrm{Li}_3(y)+\frac{1}{2}X\pi^2+\frac{1}{2}X^3-\frac{3}{2}X^2Y\bigg)\Bigg) \qquad (A.40)
\end{aligned}
$$

$$
\begin{aligned}
B^{(2),[1]}_{++++} =\; & \frac{7}{2}\frac{t}{u}X^2+\frac{t^2}{s^2}\bigg(-4\mathrm{Li}_4(x)-4\mathrm{Li}_4(y)+2\mathrm{Li}_4(z)+3Y\mathrm{Li}_3(x)+2X\mathrm{Li}_3(x)+\\
&\qquad+\mathrm{Li}_3(x)+\mathrm{Li}_3(y)+Y\mathrm{Li}_3(y)+4X\mathrm{Li}_3(y)-X\mathrm{Li}_2(x)+X^2\mathrm{Li}_2(x)-\\
&\qquad-XY\mathrm{Li}_2(x)-\frac{1}{3}\pi^2\mathrm{Li}_2(x)-Y\mathrm{Li}_2(y)-\frac{1}{2}XY^3-\frac{17}{12}XY\pi^2-
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{35}{36}X\pi^2+X^2Y^2+\frac{3}{8}X^2\pi^2+\frac{11}{24}X^2Y+\frac{1}{2}XY^2-\frac{71}{72}Y\pi^2-\\
&-\frac{29}{36}X^3+\frac{73}{18}X^2-\frac{1}{6}X^4+\frac{3}{8}Y^2\pi^2+\frac{73}{18}\pi^2+\frac{37}{120}\pi^4+\frac{1}{6}Y^4+\\
&+\frac{73}{18}Y^2-\frac{59}{72}Y^3-\frac{73}{9}XY+\frac{5}{6}X^3Y\bigg)+\\
&+\frac{t}{s}\bigg(-2\mathrm{Li}_4(x)-2\mathrm{Li}_4(y)-2\mathrm{Li}_4(z)-4\mathrm{Li}_3(x)-2X\mathrm{Li}_3(x)+6Y\mathrm{Li}_3(x)+\\
&+2\mathrm{Li}_3(y)+2Y\mathrm{Li}_3(y)+2X\mathrm{Li}_3(y)-2XY\mathrm{Li}_2(x)+2X^2\mathrm{Li}_2(x)+\\
&+\frac{1}{3}\pi^2\mathrm{Li}_2(x)+4X\mathrm{Li}_2(x)-2Y\mathrm{Li}_2(y)+2\zeta_3-\frac{25}{36}X\pi^2+\frac{1}{12}Y^4-\\
&-\frac{5}{6}XY\pi^2+\frac{1}{4}X^2\pi^2+\frac{271}{36}Y^2-\frac{17}{9}Y^3+\frac{1}{2}X^2Y^2+\frac{65}{18}X^2-\frac{1}{12}X^4+\\
&+\frac{1}{4}Y^2\pi^2+2XY^2+\frac{64}{9}X-\frac{64}{9}Y+\frac{115}{18}\pi^2+\frac{1}{12}\pi^4-\frac{401}{36}XY+\\
&+\frac{2}{3}X^3Y+\frac{8}{3}X^2Y-\frac{77}{36}Y\pi^2-\frac{19}{9}X^3\bigg)+\\
&+\bigg(3\mathrm{Li}_4(x)-3X\mathrm{Li}_3(x)+\frac{22}{3}\mathrm{Li}_3(x)+\frac{11}{3}\mathrm{Li}_3(y)-\pi^2\mathrm{Li}_2(x)-\frac{22}{3}X\mathrm{Li}_2(x)+\\
&+\frac{3}{2}X^2\mathrm{Li}_2(x)-\frac{11}{3}Y\mathrm{Li}_2(y)-\frac{1}{2}Y\zeta_3-\frac{707}{72}\zeta_3-\frac{1}{2}XY^3+\frac{7}{6}XY^2-\\
&-\frac{7}{6}XY\pi^2+\frac{599}{72}Y^2+\frac{25}{6}X\pi^2+\frac{5}{12}Y^2\pi^2+\frac{1}{8}Y^4+2X^2\pi^2-\frac{8}{3}X^2Y-\\
&-\frac{319}{144}Y\pi^2+2X^2+X^2Y^2-\frac{26}{3}X-\frac{275}{54}Y+\frac{163}{16}\pi^2-\frac{77}{1440}\pi^4\\
&+\frac{30659}{1296}-\frac{71}{36}Y^3-\frac{71}{9}XY-\frac{1}{2}X^3Y\bigg)+\\
&+i\pi\bigg(\frac{t}{s}\bigg(4\mathrm{Li}_3(x)+4\mathrm{Li}_3(y)+2X\mathrm{Li}_2(x)+6\mathrm{Li}_2(x)+2Y\mathrm{Li}_2(y)+4XY+XY^2+\\
&\qquad-\frac{1}{3}X\pi^2-\frac{1}{4}X+\frac{1}{4}Y-2X^2+3X^2Y+\frac{1}{3}Y\pi^2-\frac{2}{3}X^3-\frac{1}{6}\pi^2+Y^2\bigg)+\\
&\quad+\bigg(-3\mathrm{Li}_3(x)-Y\mathrm{Li}_2(x)+3X\mathrm{Li}_2(x)-\frac{11}{3}\mathrm{Li}_2(x)-Y\mathrm{Li}_2(y)-\frac{1}{2}\zeta_3+\\
&\qquad+\frac{3}{4}Y^2-\frac{9}{4}Y+\frac{1}{2}X^2Y-X^3+\frac{5}{6}X\pi^2+\frac{5}{6}Y\pi^2-\frac{35}{9}X+\frac{49}{54}+\\
&\qquad+\frac{83}{48}\pi^2-\frac{1}{2}XY^2-\frac{20}{3}XY+X^2\bigg)+\\
&+7\frac{t}{u}X+\frac{t^2}{s^2}\bigg(5\mathrm{Li}_3(x)+5\mathrm{Li}_3(y)+X\mathrm{Li}_2(x)+Y\mathrm{Li}_2(y)-\frac{3}{4}XY+\frac{3}{8}X^2+\frac{3}{8}Y^2+\\
&\qquad+\frac{1}{2}XY^2+\frac{1}{6}Y\pi^2-\frac{2}{3}X\pi^2+\frac{3}{8}\pi^2+3X^2Y-\frac{5}{6}X^3\bigg)\bigg)
\end{aligned}
\tag{A.41}
$$

$$
\begin{aligned}
C^{(2),[1]}_{++++}=\frac{t^2}{s^2}\bigg(&-6\mathrm{Li}_4(x)-6\mathrm{Li}_4(y)+4X\mathrm{Li}_3(x)-3\mathrm{Li}_3(x)-Y\mathrm{Li}_3(x)+5Y\mathrm{Li}_3(y)-\\
&-2X\mathrm{Li}_3(y)-3\mathrm{Li}_3(y)+3X\mathrm{Li}_2(x)-2XY\mathrm{Li}_2(x)-X^2\mathrm{Li}_2(x)+\\
&+3Y\mathrm{Li}_2(y)-2Y^2\mathrm{Li}_2(y)-XY\mathrm{Li}_2(y)-\frac{3}{2}X^2Y^2+\frac{1}{12}XY\pi^2+\\
&+\frac{1}{6}X\pi^2+\frac{5}{4}XY^2+\frac{5}{24}X^2\pi^2+\frac{11}{8}X^2Y-\frac{1}{6}X^3Y+\frac{5}{24}Y\pi^2-
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{3}X^3+\frac{3}{2}X^2-\frac{1}{12}X^4-\frac{5}{6}XY^3+\frac{5}{24}Y^2\pi^2+\frac{3}{2}\pi^2+\frac{19}{40}\pi^4+\\
&+\frac{1}{12}Y^4+\frac{3}{2}Y^2-\frac{7}{24}Y^3-3XY\bigg)+\\
&+\frac{t}{s}\bigg(-12\mathrm{Li}_4(x)-12\mathrm{Li}_4(y)-2Y\mathrm{Li}_3(x)+8X\mathrm{Li}_3(x)-2\mathrm{Li}_3(x)-4X\mathrm{Li}_3(y)-\\
&-2\mathrm{Li}_3(y)+10Y\mathrm{Li}_3(y)-2X^2\mathrm{Li}_2(x)-4XY\mathrm{Li}_2(x)+2X\mathrm{Li}_2(x)-\\
&-4Y^2\mathrm{Li}_2(y)+2Y\mathrm{Li}_2(y)-2XY\mathrm{Li}_2(y)-3X^2Y^2+\frac{1}{6}XY\pi^2+\\
&+\frac{5}{12}X\pi^2+5XY^2+\frac{5}{12}X^2\pi^2-X^2Y+\frac{1}{6}Y^4+\frac{3}{4}Y^2-\frac{1}{3}X^3+\frac{3}{2}X^2-\\
&-\frac{1}{6}X^4-\frac{5}{3}XY^3+\frac{5}{12}Y^2\pi^2+6X-6Y+\frac{5}{3}\pi^2+\frac{19}{20}\pi^4-\frac{7}{3}Y^3-\\
&-\frac{9}{4}XY-\frac{1}{3}X^3Y-\frac{23}{12}Y\pi^2\bigg)-\frac{3}{2}\frac{t}{u}X^2+\\
&+\bigg(-4\mathrm{Li}_4(x)-8\mathrm{Li}_4(z)-4X\mathrm{Li}_3(x)+8Y\mathrm{Li}_3(x)+8Y\mathrm{Li}_3(y)-8X\mathrm{Li}_3(y)+\\
&+\frac{8}{3}\pi^2\mathrm{Li}_2(x)-10XY\mathrm{Li}_2(x)+2X^2\mathrm{Li}_2(x)-4Y^2\mathrm{Li}_2(y)-2XY\mathrm{Li}_2(y)+\\
&+8X\zeta_3-5Y\zeta_3-\frac{15}{4}\zeta_3-\frac{7}{24}Y^4+\frac{59}{8}Y^2+\frac{9}{2}XY^2+\frac{17}{6}XY\pi^2+\frac{2}{3}X^3Y-\\
&-\frac{3}{2}X^2\pi^2-\frac{9}{2}X\pi^2-\frac{5}{2}Y\pi^2-\frac{13}{2}X^2Y^2-\frac{1}{6}XY^3-\frac{19}{12}Y^2\pi^2-\frac{189}{16}Y+\\
&+\frac{59}{48}\pi^2+\frac{31}{90}\pi^4+\frac{511}{64}-\frac{9}{4}Y^3-8XY\bigg)+\\
&+i\pi\bigg(\frac{t^2}{s^2}\bigg(3\mathrm{Li}_3(x)+3\mathrm{Li}_3(y)-3X\mathrm{Li}_2(x)-3Y\mathrm{Li}_2(y)-X\pi^2-\frac{1}{2}Y\pi^2+\frac{9}{4}XY-\\
&\qquad-\frac{1}{2}X^3-\frac{9}{8}X^2-\frac{3}{2}XY^2-\frac{9}{8}\pi^2-\frac{9}{8}Y^2\bigg)+\\
&\quad+\frac{t}{s}\bigg(6\mathrm{Li}_3(x)+6\mathrm{Li}_3(y)-6X\mathrm{Li}_2(x)-6Y\mathrm{Li}_2(y)+6XY-2X\pi^2-\\
&\qquad-3XY^2-\frac{5}{2}\pi^2--\frac{3}{4}Y-3X^2-3Y^2-Y\pi^2+\frac{3}{4}X-X^3\bigg)-3\frac{t}{u}X+\\
&\quad+\bigg(4\mathrm{Li}_3(x)-4X\mathrm{Li}_2(x)+3Y\mathrm{Li}_2(x)+3Y\mathrm{Li}_2(y)+3\zeta_3-\frac{5}{2}\pi^2+9XY+\\
&\qquad+\frac{27}{4}Y+X^2Y-\frac{189}{16}-8X-\frac{3}{2}X\pi^2-\frac{9}{4}Y^2-\frac{5}{2}Y\pi^2+\frac{3}{2}XY^2\bigg)\bigg)
\end{aligned}
\tag{A.42}
$$

$$
\begin{aligned}
D^{(2),[1]}_{++++} =& \bigg(\frac{1}{3}\mathrm{Li}_3(x)-\frac{1}{3}X\mathrm{Li}_2(x)-\frac{49}{36}\zeta_3-\frac{1}{6}X^2Y-\frac{1}{24}Y\pi^2+\frac{265}{108}X+\frac{11}{72}X\pi^2+\\
&+\frac{5}{18}X^3+\frac{25}{72}\pi^2-\frac{37}{36}X^2+\frac{25}{108}Y-\frac{455}{108}\bigg)+\\
&+i\pi\bigg(-\frac{1}{3}\mathrm{Li}_2(x)+\frac{1}{2}X^2-\frac{1}{3}XY-\frac{11}{18}X-\frac{11}{12}\bigg)
\end{aligned}
\tag{A.43}
$$

$$
\begin{aligned}
E^{(2),[1]}_{++++} =& \frac{t^2}{s^2}\bigg(\frac{2}{9}X\pi^2+\frac{1}{9}Y^3-\frac{1}{3}X^2Y-\frac{8}{9}Y^2+\frac{1}{9}Y\pi^2+\frac{16}{9}XY-\frac{8}{9}X^2+\frac{2}{9}X^3-\frac{8}{9}\pi^2\bigg)+\\
&+\bigg(-\frac{4}{3}\mathrm{Li}_3(x)-\frac{2}{3}\mathrm{Li}_3(y)+\frac{4}{3}X\mathrm{Li}_2(x)+\frac{2}{3}Y\mathrm{Li}_2(y)-\frac{11}{36}\zeta_3+\frac{223}{108}Y
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{3}X^2Y+\frac{2}{9}Y^3++\frac{11}{9}XY+\frac{29}{72}Y\pi^2-\frac{41}{24}\pi^2+\frac{1}{3}XY^2-\\
&-\frac{29}{18}Y^2+\frac{29}{12}X+\frac{1}{3}X\pi^2-\frac{685}{162}\Big)+\\
&+\frac{t}{s}\Big(-\frac{16}{9}X+\frac{2}{9}Y^3-\frac{7}{9}X^2-\frac{11}{9}\pi^2+\frac{16}{9}Y-\frac{2}{3}X^2Y+\frac{4}{9}X^3+\frac{4}{9}X\pi^2+\\
&\quad+\frac{2}{9}Y\pi^2-\frac{13}{9}Y^2+\frac{20}{9}XY\Big)+\\
&+i\pi\Big(\frac{2}{3}\mathrm{Li}_2(x)-\frac{1}{24}\pi^2+\frac{223}{108}+\frac{2}{3}XY-X^2+\frac{11}{9}X\Big)
\end{aligned}
\tag{A.44}
$$

$$
F^{(2);[1]}_{++++}=\left(\frac{25}{81}-\frac{10}{27}X+\frac{1}{9}X^2\right)
\tag{A.45}
$$

$$
\begin{aligned}
A^{(2),[2]}_{++++}=&\frac{t^2}{s^2}\Big(3\mathrm{Li}_4(x)+3\mathrm{Li}_4(y)+\frac{1}{2}\mathrm{Li}_4(z)-\frac{3}{2}\mathrm{Li}_3(x)-2Y\mathrm{Li}_3(x)-X\mathrm{Li}_3(x)-\\
&-\frac{3}{2}\mathrm{Li}_3(y)-2Y\mathrm{Li}_3(y)-X\mathrm{Li}_3(y)+\frac{3}{2}X\mathrm{Li}_2(x)-\frac{1}{12}\pi^2\mathrm{Li}_2(x)-\\
&-\frac{1}{4}X^2\mathrm{Li}_2(x)+\frac{1}{4}Y^2\mathrm{Li}_2(y)+\frac{3}{2}Y\mathrm{Li}_2(y)-\frac{1}{2}XY\mathrm{Li}_2(y)+\\
&+\frac{7}{12}X^2Y-\frac{7}{8}X^2Y^2+\frac{7}{12}XY\pi^2+\frac{13}{36}X\pi^2+\frac{1}{9}X^3-\frac{37}{36}X^2+\\
&+\frac{1}{24}X^4+\frac{5}{9}Y\pi^2-\frac{1}{6}Y^2\pi^2-\frac{37}{36}\pi^2-\frac{7}{36}\pi^4-\\
&-\frac{37}{36}Y^2+\frac{11}{36}Y^3+\frac{37}{18}XY-\frac{1}{6}X^3Y-\frac{1}{6}X^2\pi^2\Big)-\frac{1}{2}\frac{t}{u}X^2\\
&+\frac{t}{s}\Big(3\mathrm{Li}_4(x)+3\mathrm{Li}_4(y)+4\mathrm{Li}_4(z)+X\mathrm{Li}_3(x)-4Y\mathrm{Li}_3(x)-\frac{3}{2}\mathrm{Li}_3(x)+\\
&\quad+X\mathrm{Li}_3(y)-\frac{9}{2}\mathrm{Li}_3(y)-4Y\mathrm{Li}_3(y)-\frac{1}{2}X^2\mathrm{Li}_2(x)+\frac{3}{2}X\mathrm{Li}_2(x)-\\
&\quad-\frac{2}{3}\pi^2\mathrm{Li}_2(x)+\frac{1}{2}Y^2\mathrm{Li}_2(y)-XY\mathrm{Li}_2(y)+\frac{9}{2}Y\mathrm{Li}_2(y)+\frac{1}{2}\zeta_3-\\
&\quad-\frac{1}{12}X^2\pi^2+\frac{5}{12}X^2Y+\frac{89}{18}XY-\frac{1}{2}XY^3-\frac{95}{36}Y^2+\frac{19}{36}Y^3-X^2Y^2+\\
&\quad+\frac{1}{8}Y^4-\frac{1}{24}X^4+\frac{1}{2}XY^2+\frac{1}{6}XY\pi^2+\frac{5}{9}X\pi^2-\frac{32}{9}X-\frac{1}{12}Y^2\pi^2-\\
&\quad-\frac{22}{9}\pi^2-\frac{11}{90}\pi^4+\frac{1}{6}X^3Y+\frac{32}{9}Y+\frac{49}{36}Y\pi^2+\frac{5}{9}X^3-\frac{83}{36}X^2\Big)+\\
&+\Big(-2\mathrm{Li}_4(x)+4\mathrm{Li}_4(y)+\mathrm{Li}_4(z)+X\mathrm{Li}_3(x)-\frac{29}{6}\mathrm{Li}_3(x)-\frac{10}{3}\mathrm{Li}_3(y)-\\
&-3Y\mathrm{Li}_3(y)-2XY\mathrm{Li}_2(x)+\frac{29}{6}X\mathrm{Li}_2(x)+\frac{1}{2}X^2\mathrm{Li}_2(x)-\frac{1}{6}\pi^2\mathrm{Li}_2(x)-\\
&-XY\mathrm{Li}_2(y)+\frac{1}{2}Y^2\mathrm{Li}_2(y)+\frac{10}{3}Y\mathrm{Li}_2(y)-\frac{1}{2}X\zeta_3+4Y\zeta_3+\frac{43}{72}\zeta_3+\\
&+\frac{289}{144}X\pi^2-\frac{241}{72}Y^2-\frac{1}{6}XY\pi^2-\frac{7}{24}X^2\pi^2+\frac{11}{18}Y^3+\frac{7}{12}X^3Y+\\
&+\frac{11}{12}XY^2-\frac{9}{8}X^2Y^2+\frac{203}{72}XY-\frac{7}{12}XY^3+\frac{5}{8}Y^2\pi^2+\frac{23}{9}X+\frac{1673}{432}Y-\\
&-2\pi^2-\frac{109}{1440}\pi^4-\frac{23213}{5184}+\frac{5}{12}X^2Y+\frac{55}{72}Y\pi^2+\frac{7}{9}X^3-\frac{31}{12}X^2-\frac{1}{8}X^4\Big)+
\end{aligned}
$$

$$
\begin{aligned}
&+i\pi\Bigg(-\frac{t}{u}X+\frac{t}{s}\Bigg(-3\mathrm{Li}_3(x)-3\mathrm{Li}_3(y)-3\mathrm{Li}_2(x)+\frac{1}{6}X^3-\frac{3}{2}X-\frac{1}{2}X^2Y+\\
&\qquad+\frac{3}{2}Y-XY^2-\frac{7}{6}\pi^2-2Y^2+\frac{1}{3}Y\pi^2+\frac{1}{3}Y^3+\\
&\qquad+\frac{1}{6}X\pi^2-\frac{1}{2}X^2+XY\Bigg)+\\
&+\Bigg(\mathrm{Li}_3(x)-3\mathrm{Li}_3(y)+\frac{3}{2}\mathrm{Li}_2(x)+Y\mathrm{Li}_2(x)+3Y\mathrm{Li}_2(y)+\frac{7}{2}\zeta_3-\\
&\quad-\frac{5}{12}X^3-\frac{59}{48}\pi^2-\frac{5}{12}Y^3+\frac{409}{432}+\frac{13}{8}Y+\frac{1}{4}Y\pi^2-\frac{5}{12}X\pi^2-\frac{3}{2}X^2-\\
&\quad-\frac{3}{4}Y^2+2X^2Y+3XY-\frac{9}{8}X+\frac{1}{2}XY^2\Bigg)+\\
&+\frac{t^2}{s^2}\Bigg(-3\mathrm{Li}_3(x)-3\mathrm{Li}_3(y)-\frac{3}{4}X^2+\frac{1}{3}X^3-\frac{1}{2}XY^2-\frac{3}{4}\pi^2-X^2Y+\\
&\quad+\frac{1}{6}Y\pi^2+\frac{1}{6}Y^3+\frac{1}{3}X\pi^2-\frac{3}{4}Y^2+\frac{3}{2}XY\Bigg)\Bigg)
\end{aligned}
\tag{A.46}
$$

$$
\begin{aligned}
B^{(2),[2]}_{++++} = {}& \frac{t^2}{s^2}\Bigg(2\mathrm{Li}_4(x)+2\mathrm{Li}_4(y)+\frac{5}{2}\mathrm{Li}_3(x)-X\mathrm{Li}_3(x)+\frac{5}{2}\mathrm{Li}_3(y)-2Y\mathrm{Li}_3(y)+\\
&+X\mathrm{Li}_3(y)-\frac{5}{2}X\mathrm{Li}_2(x)+2XY\mathrm{Li}_2(x)+XY\mathrm{Li}_2(y)+Y^2\mathrm{Li}_2(y)-\\
&-\frac{5}{2}Y\mathrm{Li}_2(y)-\frac{1}{8}X^2\pi^2-\frac{3}{2}XY^2-\frac{43}{24}X^2Y+\frac{2}{3}XY^3-\frac{1}{8}Y^2\pi^2+\\
&+\frac{127}{18}XY-\frac{1}{12}XY\pi^2+\frac{4}{9}X\pi^2-\frac{127}{36}X^2+\frac{1}{24}X^4+\frac{25}{72}Y\pi^2+\\
&+\frac{31}{36}X^3-\frac{127}{36}\pi^2-\frac{31}{180}\pi^4+\frac{5}{4}X^2Y^2-\frac{1}{8}Y^4-\frac{127}{36}Y^2+\frac{55}{72}Y^3\Bigg)+\\
&+\Bigg(3\mathrm{Li}_4(x)-4\mathrm{Li}_4(y)+4\mathrm{Li}_4(z)+3X\mathrm{Li}_3(x)-\frac{2}{3}\mathrm{Li}_3(x)-5Y\mathrm{Li}_3(x)-\\
&-\frac{1}{3}\mathrm{Li}_3(y)+5X\mathrm{Li}_3(y)-2Y\mathrm{Li}_3(y)+8XY\mathrm{Li}_2(x)-\pi^2\mathrm{Li}_2(x)-\\
&-\frac{5}{2}X^2\mathrm{Li}_2(x)+\frac{2}{3}X\mathrm{Li}_2(x)+2XY\mathrm{Li}_2(y)+2Y^2\mathrm{Li}_2(y)+\frac{1}{3}Y\mathrm{Li}_2(y)-\\
&-5X\zeta_3+\frac{467}{72}\zeta_3+5Y\zeta_3-\frac{1}{8}X^2\pi^2-\frac{5}{6}XY\pi^2-\frac{7}{12}X\pi^2-\frac{83}{9}Y^2+\\
&+\frac{4}{3}X^2Y+\frac{653}{72}XY+\frac{1}{24}Y^4+\frac{5}{4}XY^3+\frac{103}{36}Y^3+\frac{31}{8}X^2Y^2+\frac{26}{3}X+\\
&+\frac{49}{6}Y-\frac{1}{24}Y^2\pi^2-\frac{53}{480}\pi^4-\frac{30659}{1296}-\frac{1}{4}X^3Y-\frac{751}{144}\pi^2-\\
&-\frac{61}{12}XY^2+\frac{49}{18}Y\pi^2-2X^2\Bigg)-\frac{1}{2}\frac{t}{u}X^2+\\
&+\frac{t}{s}\Bigg(4\mathrm{Li}_4(x)+4\mathrm{Li}_4(y)+4\mathrm{Li}_3(x)-2X\mathrm{Li}_3(x)+2X\mathrm{Li}_3(y)-4Y\mathrm{Li}_3(y)+\\
&+4\mathrm{Li}_3(y)-4X\mathrm{Li}_2(x)+4XY\mathrm{Li}_2(x)+2XY\mathrm{Li}_2(y)-4Y\mathrm{Li}_2(y)+\\
&+2Y^2\mathrm{Li}_2(y)-6XY^2-\frac{1}{6}XY\pi^2-\frac{1}{4}Y^4+\frac{4}{3}XY^3-\frac{1}{4}Y^2\pi^2-\frac{1}{4}X^2\pi^2-\\
&-\frac{5}{6}X^2Y+\frac{11}{36}X\pi^2-\frac{65}{36}X^2+\frac{1}{12}X^4+\frac{5}{2}X^2Y^2+\frac{11}{9}X^3-\frac{59}{9}X+\\
&+\frac{59}{9}Y-\frac{71}{18}\pi^2-\frac{31}{90}\pi^4-\frac{44}{9}Y^2+\frac{53}{18}Y^3+\frac{241}{36}XY+\frac{73}{36}Y\pi^2\Bigg)+
\end{aligned}
$$

$$
\begin{aligned}
&+i\pi\Bigg(-\frac{t}{u}X+\Bigg(-2\mathrm{Li}_3(x)+3\mathrm{Li}_3(y)+X\mathrm{Li}_2(x)-2Y\mathrm{Li}_2(x)+\frac{1}{3}\mathrm{Li}_2(x)-\\
&\qquad -4Y\mathrm{Li}_2(y)-\frac{25}{6}XY+\frac{365}{72}X+\frac{11}{12}X^3-XY^2+\frac{7}{12}Y\pi^2+\\
&\qquad +\frac{3}{2}Y^2+\frac{5}{12}Y^3+\frac{5}{4}\pi^2+\frac{13}{6}-\frac{5}{2}X^2Y+X^2-\frac{31}{8}Y+\frac{3}{4}X\pi^2\Bigg)+\\
&\quad +\frac{t}{s}\Bigg(-2\mathrm{Li}_3(x)-2\mathrm{Li}_3(y)+2X\mathrm{Li}_2(x)+2Y\mathrm{Li}_2(y)-\frac{5}{4}Y-6XY+\frac{2}{3}X^3+\\
&\qquad +\frac{5}{2}\pi^2+X\pi^2+2XY^2+3Y^2+\frac{5}{4}X+3X^2-X^2Y-\frac{1}{3}Y^3\Bigg)+\\
&\quad +\frac{t^2}{s^2}\Bigg(-\mathrm{Li}_3(x)-\mathrm{Li}_3(y)+X\mathrm{Li}_2(x)+Y\mathrm{Li}_2(y)+\frac{9}{8}\pi^2+\frac{9}{8}Y^2+\frac{1}{3}X^3-\frac{1}{6}Y^3+\\
&\qquad +\frac{9}{8}X^2+XY^2+\frac{1}{2}X\pi^2-\frac{1}{2}X^2Y-\frac{9}{4}XY\Bigg)\Bigg)
\end{aligned}
\tag{A.47}
$$

$$
\begin{aligned}
C^{(2),[2]}_{++++} =\; &\frac{t^2}{s^2}\Bigg(2\mathrm{Li}_4(x)+2\mathrm{Li}_4(y)+\frac{1}{2}\mathrm{Li}_4(z)+\mathrm{Li}_3(x)-X\mathrm{Li}_3(x)+\mathrm{Li}_3(y)-2Y\mathrm{Li}_3(y)+\\
&\quad +X\mathrm{Li}_3(y)-X\mathrm{Li}_2(x)+\frac{1}{4}X^2\mathrm{Li}_2(x)+XY\mathrm{Li}_2(x)-\frac{1}{12}\pi^2\mathrm{Li}_2(x)+\\
&\quad +\frac{1}{2}XY\mathrm{Li}_2(y)-Y\mathrm{Li}_2(y)+\frac{3}{4}Y^2\mathrm{Li}_2(y)-\frac{1}{4}XY^2-\frac{1}{24}Y^2\pi^2-\\
&\quad -\frac{5}{8}X^2Y-\frac{1}{6}XY\pi^2+\frac{1}{6}XY^3-X^2+\frac{1}{24}X^4-\frac{1}{8}Y\pi^2+\frac{1}{6}X^3-\pi^2-\\
&\quad -\frac{3}{20}\pi^4+\frac{7}{8}X^2Y^2-Y^2+\frac{1}{24}Y^3+2XY-\frac{1}{24}X^2\pi^2\Bigg)+\\
&+\Bigg(\mathrm{Li}_4(x)+3\mathrm{Li}_4(z)-3Y\mathrm{Li}_3(x)+2X\mathrm{Li}_3(x)+3X\mathrm{Li}_3(y)-3Y\mathrm{Li}_3(y)+\\
&\quad +4XY\mathrm{Li}_2(x)-\frac{5}{6}\pi^2\mathrm{Li}_2(x)-X^2\mathrm{Li}_2(x)+\frac{3}{2}Y^2\mathrm{Li}_2(y)+XY\mathrm{Li}_2(y)\\
&\quad +\frac{15}{4}\zeta_3-3X\zeta_3+\frac{1}{2}X^2\pi^2+\frac{1}{8}Y^4+\frac{7}{12}Y^2\pi^2+\frac{3}{2}X\pi^2-\frac{1}{3}X^3Y-\\
&\quad -XY\pi^2+Y\pi^2+4XY-\frac{3}{2}XY^2+\frac{11}{4}X^2Y^2+\frac{93}{16}Y-\frac{71}{48}\pi^2-\frac{17}{180}\pi^4-\\
&\quad -\frac{511}{64}-\frac{25}{8}Y^2+\frac{3}{4}Y^3\Bigg)+\frac{1}{2}\frac{t}{u}X^2+\\
&+\frac{t}{s}\Bigg(4\mathrm{Li}_4(x)+4\mathrm{Li}_4(y)+\mathrm{Li}_4(z)+\frac{1}{2}\mathrm{Li}_3(x)-2X\mathrm{Li}_3(x)+2X\mathrm{Li}_3(y)-\\
&\quad -4Y\mathrm{Li}_3(y)+\frac{1}{2}\mathrm{Li}_3(y)+\frac{1}{2}X^2\mathrm{Li}_2(x)+2XY\mathrm{Li}_2(x)-\frac{1}{6}\pi^2\mathrm{Li}_2(x)-\\
&\quad -\frac{1}{2}X\mathrm{Li}_2(x)-\frac{1}{2}Y\mathrm{Li}_2(y)+XY\mathrm{Li}_2(y)+\frac{3}{2}Y^2\mathrm{Li}_2(y)+\frac{1}{2}\zeta_3-\frac{1}{12}X\pi^2-\\
&\quad -\frac{3}{4}Y^2-\frac{3}{2}XY^2-\frac{1}{3}XY\pi^2+\frac{1}{4}X^2Y-\frac{1}{12}Y^2\pi^2+\frac{1}{12}X^4+\\
&\quad +\frac{7}{4}X^2Y^2+\frac{3}{4}Y^3+\frac{1}{3}XY^3-3X+3Y-\frac{7}{6}\pi^2-\frac{3}{10}\pi^4+\\
&\quad +\frac{7}{4}XY-\frac{1}{12}X^2\pi^2+\frac{2}{3}Y\pi^2+\frac{1}{6}X^3-X^2\Bigg)+\\
&+i\pi\Bigg(\frac{t^2}{s^2}\Bigg(-\mathrm{Li}_3(x)-\mathrm{Li}_3(y)+X\mathrm{Li}_2(x)+Y\mathrm{Li}_2(y)+\frac{3}{8}\pi^2-\frac{3}{4}XY+\frac{1}{3}X\pi^2+
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{3}{8}X^2 + \frac{1}{6}X^3 + \frac{1}{6}Y\pi^2 + \frac{1}{2}XY^2 + \frac{3}{8}Y^2\Bigg) + \\
& + \frac{t}{s}\Bigg(-2\mathrm{Li}_3(x) - 2\mathrm{Li}_3(y) + 2X\mathrm{Li}_2(x) + 2Y\mathrm{Li}_2(y) + \frac{2}{3}X\pi^2 + X^2 + \frac{5}{6}\pi^2 + \\
& + \frac{1}{4}Y + \frac{1}{3}X^3 + XY^2 + Y^2 - \frac{1}{4}X + \frac{1}{3}Y\pi^2 - 2XY\Bigg) + \frac{t}{u}X + \\
& + \Bigg(-\mathrm{Li}_3(x) + X\mathrm{Li}_2(x) - Y\mathrm{Li}_2(x) - Y\mathrm{Li}_2(y) - 3\zeta_3 + 4X + \pi^2 - \frac{9}{4}Y + \\
& + \frac{1}{2}X\pi^2 - \frac{1}{2}X^2Y + \frac{5}{6}Y\pi^2 + \frac{93}{16} - \frac{1}{2}XY^2 - 3XY + \frac{3}{4}Y^2\Bigg)\Bigg)
\end{aligned}
\tag{A.48}
$$

$$
\begin{aligned}
D^{(2),[2]}_{++++} = & \frac{t^2}{s^2}\Bigg(\frac{4}{9}X^2 + \frac{4}{9}Y^2 - \frac{1}{18}Y^3 - \frac{1}{18}Y\pi^2 + \frac{1}{6}X^2Y + \frac{4}{9}\pi^2 - \frac{1}{9}X^3 - \frac{1}{9}X\pi^2 - \frac{8}{9}XY\Bigg) + \\
& + \frac{t}{s}\Bigg(-\frac{2}{9}X^3 + \frac{7}{18}X^2 - \frac{10}{9}XY + \frac{13}{18}Y^2 + \frac{8}{9}X + \frac{1}{3}X^2Y - \frac{1}{9}Y\pi^2 - \frac{8}{9}Y - \\
& - \frac{1}{9}Y^3 - \frac{2}{9}X\pi^2 + \frac{11}{18}\pi^2\Bigg) + \\
& + \Bigg(\frac{1}{3}\mathrm{Li}_3(x) + \frac{1}{3}\mathrm{Li}_3(y) - \frac{1}{3}X\mathrm{Li}_2(x) - \frac{1}{3}Y\mathrm{Li}_2(y) + \frac{37}{36}\zeta_3 + \frac{37}{36}X^2 - \frac{149}{108}Y - \\
& - \frac{23}{72}X\pi^2 - \frac{11}{18}XY + \frac{29}{36}Y^2 - \frac{1}{6}XY^2 + \frac{11}{24}\pi^2 - \frac{5}{18}X^3 + \frac{1}{3}X^2Y - \\
& - \frac{265}{108}X + \frac{455}{108} - \frac{1}{9}Y^3 - \frac{5}{36}Y\pi^2\Bigg) + \\
& + i\pi\Bigg(-\frac{25}{108} + \frac{1}{24}\pi^2\Bigg)
\end{aligned}
\tag{A.49}
$$

$$
\begin{aligned}
E^{(2),[2]}_{++++} = & \Bigg(\frac{2}{3}\mathrm{Li}_3(x) + \frac{1}{3}\mathrm{Li}_3(y) - \frac{2}{3}X\mathrm{Li}_2(x) - \frac{1}{3}Y\mathrm{Li}_2(y) + \frac{23}{36}\zeta_3 + \frac{1}{6}X^2Y + \frac{29}{36}Y^2 - \\
& - \frac{11}{18}XY - \frac{29}{12}X + \frac{685}{162} - \frac{1}{6}XY^2 - \frac{2}{9}Y\pi^2 - \frac{11}{12}Y - \frac{1}{9}Y^3 + \frac{65}{72}\pi^2 - \frac{1}{6}X\pi^2\Bigg) + \\
& + \frac{t^2}{s^2}\Bigg(\frac{4}{9}X^2 + \frac{4}{9}Y^2 - \frac{1}{18}Y^3 - \frac{1}{18}Y\pi^2 + \frac{1}{6}X^2Y + \frac{4}{9}\pi^2 - \frac{1}{9}X^3 - \frac{1}{9}X\pi^2 - \frac{8}{9}XY\Bigg) + \\
& + \frac{t}{s}\Bigg(-\frac{2}{9}X^3 + \frac{7}{18}X^2 - \frac{10}{9}XY + \frac{13}{18}Y^2 + \frac{8}{9}X + \frac{1}{3}X^2Y - \frac{1}{9}Y\pi^2 - \frac{8}{9}Y - \\
& - \frac{1}{9}Y^3 - \frac{2}{9}X\pi^2 + \frac{11}{18}\pi^2\Bigg) + \\
& + i\pi\Bigg(-\frac{1}{3}\mathrm{Li}_2(x) + \frac{1}{2}X^2 - \frac{1}{3}XY - \frac{11}{18}X - \frac{11}{12}\Bigg)
\end{aligned}
\tag{A.50}
$$

$$
F^{(2),[2]}_{++++} = \Bigg(\frac{10}{27}X - \frac{1}{9}X^2 - \frac{25}{81}\Bigg)
\tag{A.51}
$$

A.5 $q(p_2,+) + Q(p_1,+) \to Q(p_3,+) + q(p_4,+)$

$$
\begin{aligned}
A^{(2),[1]}_{u++++} = & \frac{t^2}{s^2}\Bigg(\frac{3}{2}\mathrm{Li}_4(z) - Y\mathrm{Li}_3(x) + X\mathrm{Li}_3(x) - Y\mathrm{Li}_3(y) + X\mathrm{Li}_3(y) - \frac{1}{4}X^2\mathrm{Li}_2(x) + \\
& + XY\mathrm{Li}_2(x) - \frac{1}{4}\pi^2\mathrm{Li}_2(x) + \frac{1}{4}Y^2\mathrm{Li}_2(y) + \frac{1}{2}XY\mathrm{Li}_2(y) + \frac{11}{240}\pi^4 + \frac{16}{9}\pi^2 +
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{5}{8}X^2Y - \frac{5}{12}XY^3 + \frac{1}{8}X^2\pi^2 - \frac{29}{72}Y\pi^2 + \frac{1}{8}Y^2\pi^2 + \frac{5}{4}X^2Y^2 - \\
& - \frac{1}{2}XY\pi^2 - \frac{1}{4}X^3Y - \frac{32}{9}XY + \frac{7}{24}XY^2 - \frac{37}{72}X\pi^2 + \frac{5}{48}Y^4 - \\
& - \frac{37}{72}X^3 + \frac{1}{16}X^4 + \frac{16}{9}X^2 + \frac{16}{9}Y^2 - \frac{29}{72}Y^3\Bigg) + \\
& + \frac{t}{s}\Bigg(-\frac{1}{2}\mathrm{Li}_3(x) - \frac{1}{2}\mathrm{Li}_3(y) + \frac{1}{2}X\mathrm{Li}_2(x) + \frac{1}{2}Y\mathrm{Li}_2(y) + \frac{3}{2}\zeta_3 - \frac{13}{6}XY + \\
& + XY^2 - \frac{3}{4}X^2Y + \frac{1}{6}X^2 - \frac{1}{4}Y^3 + \frac{32}{9}X - \frac{32}{9}Y + 2Y^2 + \frac{43}{36}\pi^2 + \\
& + \frac{1}{3}X\pi^2 + \frac{1}{3}X^3 - \frac{1}{12}Y\pi^2\Bigg) + \\
& + \Bigg(-\frac{1}{2}\mathrm{Li}_4(z) + \frac{4}{3}\mathrm{Li}_3(x) + \frac{4}{3}\mathrm{Li}_3(y) + \frac{1}{12}\pi^2\mathrm{Li}_2(x) + XY\mathrm{Li}_2(x) - \frac{1}{4}X^2\mathrm{Li}_2(x) + \\
& - \frac{4}{3}X\mathrm{Li}_2(x) + \frac{1}{4}Y^2\mathrm{Li}_2(y) + \frac{1}{2}XY\mathrm{Li}_2(y) - \frac{4}{3}Y\mathrm{Li}_2(y) + 2X\zeta_3 + \frac{173}{72}\zeta_3 + \\
& + \frac{3}{2}Y\zeta_3 + \frac{47}{1440}\pi^4 + \frac{23213}{5184} + \frac{17}{12}\pi^2 - \frac{15}{16}Y\pi^2 - \frac{1}{12}XY^3 + \frac{227}{144}X - \frac{55}{9}Y - \\
& - \frac{1}{4}X^3Y + \frac{1}{8}Y^2\pi^2 + X^2Y^2 - \frac{1}{24}X^2\pi^2 - \frac{1}{6}XY\pi^2 - \frac{23}{9}XY + \frac{7}{24}XY^2 - \\
& - \frac{7}{8}X\pi^2 - \frac{1}{24}X^2Y + \frac{1}{16}Y^4 - \frac{37}{72}X^3 + \frac{1}{48}X^4 + \frac{28}{9}X^2 + \frac{101}{36}Y^2 - \frac{5}{8}Y^3\Bigg) + \\
& + i\pi\left(\frac{7}{2}\zeta_3 + \frac{409}{432} - \frac{11}{48}\pi^2\right)
\end{aligned}
\tag{A.52}
$$

$$
\begin{aligned}
B^{(2),[1]}_{u++++} = & -\frac{5}{2}\frac{u}{t}Y^2 + \frac{t}{s}\Bigg(6\mathrm{Li}_4(x) + 6\mathrm{Li}_4(y) + 6\mathrm{Li}_4(z) - 6Y\mathrm{Li}_3(x) + \frac{3}{2}\mathrm{Li}_3(x) - \frac{9}{2}\mathrm{Li}_3(y) - \\
& - 6Y\mathrm{Li}_3(y) - \pi^2\mathrm{Li}_2(x) - \frac{3}{2}X\mathrm{Li}_2(x) + \frac{9}{2}Y\mathrm{Li}_2(y) + \frac{11}{2}\zeta_3 - \frac{37}{9}X + \\
& + \frac{1}{9}\pi^2 - \frac{3}{4}X\pi^2 + \frac{37}{9}Y + \frac{7}{12}XY + XY^2 - \frac{1}{4}\pi^4 + \frac{3}{2}X^2Y + \frac{11}{12}Y\pi^2 - \\
& - 2XY^3 + \frac{1}{2}Y^4 - \frac{5}{6}X^3 + \frac{17}{12}X^2 - 2Y^2 - \frac{2}{3}Y^3\Bigg) + \\
& + \frac{t^2}{s^2}\Bigg(8\mathrm{Li}_4(x) + 8\mathrm{Li}_4(y) + \frac{11}{2}\mathrm{Li}_4(z) - 5Y\mathrm{Li}_3(x) + \mathrm{Li}_3(x) - 2X\mathrm{Li}_3(x) + \mathrm{Li}_3(y) - \\
& - 7Y\mathrm{Li}_3(y) - 2XY\mathrm{Li}_2(x) - X\mathrm{Li}_2(x) - \frac{11}{12}\pi^2\mathrm{Li}_2(x) + \frac{3}{4}X^2\mathrm{Li}_2(x) - Y\mathrm{Li}_2(y) - \\
& - \frac{3}{2}XY\mathrm{Li}_2(y) + \frac{1}{4}Y^2\mathrm{Li}_2(y) - \frac{61}{144}\pi^4 - \frac{5}{4}XY^3 - \frac{14}{9}\pi^2 - \frac{1}{4}X^2\pi^2 + \frac{17}{36}Y\pi^2 - \\
& - \frac{1}{4}Y^2\pi^2 - \frac{11}{4}X^2Y^2 + \frac{3}{4}XY\pi^2 + \frac{11}{12}X^3Y + \frac{28}{9}XY - \frac{5}{24}XY^2 + \frac{89}{72}X\pi^2 - \\
& - \frac{5}{2}X^2Y + \frac{17}{48}Y^4 + \frac{101}{72}X^3 - \frac{3}{16}X^4 - \frac{14}{9}X^2 - \frac{14}{9}Y^2 + \frac{23}{36}Y^3\Bigg) + \\
& + \Bigg(-2\mathrm{Li}_4(x) - 2\mathrm{Li}_4(y) - \frac{5}{2}\mathrm{Li}_4(z) + 2Y\mathrm{Li}_3(x) - X\mathrm{Li}_3(x) - \frac{19}{6}\mathrm{Li}_3(x) - 3X\mathrm{Li}_3(y) - \\
& - \frac{77}{6}\mathrm{Li}_3(y) + 4Y\mathrm{Li}_3(y) + \frac{5}{12}\pi^2\mathrm{Li}_2(x) + \frac{19}{6}X\mathrm{Li}_2(x) + \frac{3}{4}X^2\mathrm{Li}_2(x) - 5XY\mathrm{Li}_2(x) + \\
& + \frac{77}{6}Y\mathrm{Li}_2(y) - \frac{7}{4}Y^2\mathrm{Li}_2(y) - \frac{5}{2}XY\mathrm{Li}_2(y) + \frac{7}{2}X\zeta_3 + \frac{217}{72}\zeta_3 - 3Y\zeta_3 - \frac{41}{9}Y - \\
& - \frac{271}{1440}\pi^4 + \frac{30659}{1296} - \frac{725}{144}\pi^2 + \frac{1}{4}X^2\pi^2 + \frac{7}{12}X^3Y - Y^2\pi^2 + \frac{19}{12}XY\pi^2 - \frac{271}{54}X +
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{49}{36}XY+\frac{31}{8}XY^2+\frac{83}{48}X\pi^2-\frac{1}{6}X^2Y-\frac{13}{18}Y\pi^2+\frac{1}{4}XY^3-\frac{19}{4}X^2Y^2+\frac{1}{48}Y^4+\\
&+\frac{89}{72}X^3-\frac{1}{16}X^4-\frac{403}{72}X^2+\frac{14}{9}Y^2-\frac{47}{36}Y^3\Big)+\\
&+i\pi\Bigg(\frac{t^2}{s^2}\Big(-7\mathrm{Li}_3(x)-7\mathrm{Li}_3(y)+X\mathrm{Li}_2(x)+Y\mathrm{Li}_2(y)+\frac{1}{2}X^2Y+\frac{3}{8}\pi^2+\frac{4}{3}Y\pi^2+\\
&\quad+\frac{3}{8}X^2+\frac{3}{8}Y^2+\frac{7}{6}Y^3+\frac{1}{6}X\pi^2-\frac{3}{4}XY-3XY^2\Big)-5\frac{u}{t}Y+\\
&\quad+\frac{t}{s}\Big(-6\mathrm{Li}_3(x)-6\mathrm{Li}_3(y)-6\mathrm{Li}_2(x)-3XY^2-\frac{1}{4}X^2+Y\pi^2-\frac{1}{4}X+\frac{1}{2}XY-\\
&\quad-\frac{1}{12}\pi^2-\frac{13}{4}Y^2+Y^3+\frac{1}{4}Y\Big)+ \qquad\qquad (A.53)\\
&\quad+\Big(\mathrm{Li}_3(x)+\mathrm{Li}_3(y)-X\mathrm{Li}_2(x)-\frac{29}{3}\mathrm{Li}_2(x)-Y\mathrm{Li}_2(y)+\frac{1}{2}\zeta_3+\frac{275}{54}+\frac{1}{8}X^2-\\
&\quad-\frac{5}{2}X+\frac{5}{6}Y^3-\frac{11}{144}\pi^2-\frac{7}{4}XY-XY^2-\frac{37}{8}Y^2+\frac{293}{36}Y+\frac{1}{2}X\pi^2-\frac{1}{6}Y\pi^2\Big)\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
C^{(2),[1]}_{u++++}=\;&\frac{t}{s}\Big(-2\mathrm{Li}_3(x)-2\mathrm{Li}_3(y)+2X\mathrm{Li}_2(x)+2Y\mathrm{Li}_2(y)-6\zeta_3-\frac{10}{3}\pi^2-6X+6Y+\frac{3}{4}XY^2+\\
&+\frac{1}{3}Y^3+\frac{33}{4}XY-\frac{1}{2}X^2Y+\frac{7}{12}Y\pi^2+\frac{2}{3}X\pi^2-\frac{15}{4}X^2+\frac{3}{4}X^3-\frac{9}{2}Y^2\Big)+\\
&+\frac{t^2}{s^2}\Big(-6\mathrm{Li}_4(x)-6\mathrm{Li}_4(y)-6\mathrm{Li}_4(z)+2Y\mathrm{Li}_3(x)-3\mathrm{Li}_3(x)+X\mathrm{Li}_3(x)-5X\mathrm{Li}_3(y)-\\
&\quad-3\mathrm{Li}_3(y)+8Y\mathrm{Li}_3(y)+\frac{1}{2}XY\mathrm{Li}_2(x)+\pi^2\mathrm{Li}_2(x)+3X\mathrm{Li}_2(x)-X^2\mathrm{Li}_2(x)+\\
&\quad+\frac{3}{2}XY\mathrm{Li}_2(y)+3Y\mathrm{Li}_2(y)-2Y^2\mathrm{Li}_2(y)+\frac{1}{6}XY^3-\frac{9}{2}\pi^2+\frac{1}{24}X^2\pi^2+\frac{7}{24}\pi^4+\\
&\quad+\frac{1}{24}Y^2\pi^2+X^2Y^2+\frac{1}{2}XY\pi^2-\frac{7}{6}X^3Y+9XY-\frac{5}{8}XY^2-\frac{11}{24}X\pi^2+\\
&\quad+\frac{13}{4}X^2Y+\frac{5}{6}Y\pi^2-\frac{1}{6}Y^4-\frac{23}{24}X^3+\frac{1}{6}X^4-\frac{9}{2}X^2-\frac{9}{2}Y^2+\frac{1}{3}Y^3\Big)-\frac{3}{2}\frac{u}{t}Y^2+\\
&+\Big(6\mathrm{Li}_4(x)-2\mathrm{Li}_4(y)+10\mathrm{Li}_4(z)-6Y\mathrm{Li}_3(x)+3X\mathrm{Li}_3(x)+\mathrm{Li}_3(x)+\mathrm{Li}_3(y)-4Y\mathrm{Li}_3(y)+\\
&+9X\mathrm{Li}_3(y)-\frac{13}{3}\pi^2\mathrm{Li}_2(x)+\frac{19}{2}XY\mathrm{Li}_2(x)-X\mathrm{Li}_2(x)-X^2\mathrm{Li}_2(x)-Y\mathrm{Li}_2(y)+\\
&+\frac{9}{2}XY\mathrm{Li}_2(y)-7X\zeta_3+4Y\zeta_3-\frac{39}{4}\zeta_3+6Y+\frac{247}{360}\pi^4+\frac{511}{64}-\frac{269}{48}\pi^2-\frac{5}{8}X^2\pi^2-\\
&-\frac{1}{3}X^3Y-\frac{29}{24}Y^2\pi^2+\frac{17}{2}X^2Y^2-\frac{11}{3}XY\pi^2+\frac{93}{16}X+\frac{29}{4}XY+\frac{11}{8}XY^2-\frac{7}{8}X\pi^2+\\
&+\frac{3}{4}X^2Y-\frac{19}{4}Y\pi^2-\frac{3}{2}XY^3+\frac{1}{3}Y^4-\frac{13}{24}X^3+\frac{1}{24}X^4+\frac{1}{8}X^2\Big)+\\
&+i\pi\Bigg(\frac{t^2}{s^2}\Big(3\mathrm{Li}_3(x)+3\mathrm{Li}_3(y)-3X\mathrm{Li}_2(x)-3Y\mathrm{Li}_2(y)-\frac{9}{8}X^2-Y\pi^2-\frac{9}{8}Y^2+\\
&\quad+\frac{9}{4}XY-\frac{3}{2}X^2Y-\frac{9}{8}\pi^2-\frac{1}{2}Y^3-\frac{1}{2}X\pi^2\Big)+\\
&\quad+\frac{t}{s}\Big(-\frac{3}{2}XY+\frac{3}{4}X^2+\frac{3}{4}Y^2+\frac{3}{4}X-\frac{3}{4}Y+\frac{1}{4}\pi^2\Big)-3\frac{u}{t}Y\\
&\quad+\Big(-3\mathrm{Li}_3(x)+5\mathrm{Li}_3(y)+3X\mathrm{Li}_2(x)-2Y\mathrm{Li}_2(x)-7Y\mathrm{Li}_2(y)-3\zeta_3-
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3}{8}X^2-\frac{3}{2}X\pi^2+\frac{189}{16}+\frac{29}{4}Y+\frac{15}{8}Y^2-\frac{1}{6}Y\pi^2+\frac{15}{2}X-3XY^2-\\
& -\frac{5}{8}\pi^2+\frac{21}{4}XY+\frac{1}{2}Y^3\Bigg)\Bigg)
\end{aligned}
\tag{A.54}
$$

$$
\begin{aligned}
D^{(2),[1]}_{u++++} =\ & \frac{t}{s}\left(-\frac{1}{2}Y^2+\frac{8}{9}Y-\frac{5}{18}\pi^2-\frac{1}{6}X^2-\frac{8}{9}X+\frac{2}{3}XY\right)+\\
& +\frac{t^2}{s^2}\left(\frac{1}{9}Y\pi^2-\frac{4}{9}\pi^2-\frac{4}{9}X^2+\frac{1}{9}Y^3+\frac{8}{9}XY-\frac{4}{9}Y^2+\frac{1}{18}X^3+\frac{1}{18}X\pi^2-\frac{1}{6}XY^2\right)+\\
& +\Bigg(-\frac{1}{3}\mathrm{Li}_3(x)-\frac{1}{3}\mathrm{Li}_3(y)+\frac{1}{3}X\mathrm{Li}_2(x)+\frac{1}{3}Y\mathrm{Li}_2(y)-\frac{37}{36}\zeta_3-\frac{13}{12}Y^2+\frac{1}{18}X^3-\\
& \quad -\frac{7}{24}\pi^2+\frac{5}{24}Y\pi^2+\frac{361}{108}Y+\frac{1}{6}X\pi^2+\frac{1}{6}Y^3+\frac{7}{18}XY+\frac{1}{36}X-\frac{1}{6}XY^2-\\
& \quad -\frac{455}{108}+\frac{1}{6}X^2Y-\frac{19}{36}X^2\Bigg)+i\pi\left(-\frac{25}{108}+\frac{1}{24}\pi^2\right)
\end{aligned}
\tag{A.55}
$$

$$
\begin{aligned}
E^{(2),[1]}_{u++++} =\ & \frac{t}{s}\left(-\frac{4}{3}XY-\frac{16}{9}Y+Y^2+\frac{1}{3}X^2+\frac{5}{9}\pi^2+\frac{16}{9}X\right)+\\
& +\frac{t^2}{s^2}\left(\frac{1}{3}XY^2+\frac{8}{9}\pi^2-\frac{1}{9}X^3+\frac{8}{9}Y^2-\frac{2}{9}Y\pi^2-\frac{16}{9}XY+\frac{8}{9}X^2-\frac{2}{9}Y^3-\frac{1}{9}X\pi^2\right)+\\
& +\Bigg(\frac{2}{3}\mathrm{Li}_3(x)+\frac{4}{3}\mathrm{Li}_3(y)-\frac{2}{3}X\mathrm{Li}_2(x)-\frac{4}{3}Y\mathrm{Li}_2(y)-\frac{59}{36}\zeta_3+\frac{23}{36}Y-\frac{7}{9}XY-\frac{1}{3}X^2Y-\\
& \quad -\frac{31}{108}X-\frac{1}{9}Y\pi^2+\frac{1}{9}Y^2-\frac{1}{9}X^3+\frac{2}{9}Y^3-\frac{7}{24}X\pi^2+\frac{19}{18}X^2+\frac{85}{72}\pi^2-\frac{685}{162}\Bigg)+\\
& +i\pi\left(\frac{2}{3}\mathrm{Li}_2(x)-\frac{5}{72}\pi^2-\frac{11}{9}Y-\frac{223}{108}+Y^2\right)
\end{aligned}
\tag{A.56}
$$

$$
F^{(2),[1]}_{u++++} = \left(\frac{1}{9}Y^2+\frac{25}{81}-\frac{10}{27}Y\right)
\tag{A.57}
$$

$$
\begin{aligned}
A^{(2),[2]}_{u++++} =\ & \frac{t^2}{s^2}\Bigg(-8\mathrm{Li}_4(x)-8\mathrm{Li}_4(y)-5\mathrm{Li}_4(z)+5Y\mathrm{Li}_3(x)+2X\mathrm{Li}_3(x)+\frac{1}{2}\mathrm{Li}_3(x)+\frac{1}{2}\mathrm{Li}_3(y)+\\
& \quad +7Y\mathrm{Li}_3(y)+\frac{5}{6}\pi^2\mathrm{Li}_2(x)-\frac{1}{2}X\mathrm{Li}_2(x)-\frac{1}{2}X^2\mathrm{Li}_2(x)-\frac{1}{2}Y\mathrm{Li}_2(y)-\frac{1}{2}Y^2\mathrm{Li}_2(y)-\\
& \quad -\frac{5}{4}\pi^2+\frac{17}{40}\pi^4+\frac{3}{2}XY^3+\frac{1}{8}X^2\pi^2+\frac{5}{24}Y\pi^2+\frac{1}{8}Y^2\pi^2+\frac{1}{4}X^2Y^2-\frac{1}{4}XY\pi^2-\\
& \quad -\frac{1}{6}X^3Y+\frac{5}{2}XY-\frac{3}{4}XY^2-\frac{1}{12}X\pi^2+\frac{1}{8}X^2Y-\frac{5}{12}Y^4-\frac{5}{4}X^2-\frac{5}{4}Y^2+\frac{7}{24}Y^3\Bigg)+\\
& +\frac{3}{2}\frac{u}{t}Y^2+\frac{t}{s}\Bigg(-6\mathrm{Li}_4(x)-6\mathrm{Li}_4(y)-6\mathrm{Li}_4(z)+6Y\mathrm{Li}_3(x)-2\mathrm{Li}_3(x)+4\mathrm{Li}_3(y)+\\
& \qquad +6Y\mathrm{Li}_3(y)+2X\mathrm{Li}_2(x)+\pi^2\mathrm{Li}_2(x)-4Y\mathrm{Li}_2(y)-5\zeta_3-X^2-\frac{11}{4}Y^2+\\
& \qquad +\frac{3}{4}X^2Y+\frac{13}{12}Y^3-\frac{5}{2}XY^2+\frac{15}{4}XY+\frac{5}{12}X\pi^2-\frac{29}{12}\pi^2+\frac{1}{4}\pi^4-\\
& \qquad -\frac{1}{2}Y\pi^2-\frac{1}{2}Y^4+2XY^3\Bigg)+\\
& +\Bigg(-2\mathrm{Li}_4(x)+3\mathrm{Li}_4(y)+\mathrm{Li}_4(z)+2X\mathrm{Li}_3(x)-\mathrm{Li}_3(x)+X\mathrm{Li}_3(y)+\frac{25}{3}\mathrm{Li}_3(y)-\\
& \quad -2Y\mathrm{Li}_3(y)+X\mathrm{Li}_2(x)-\frac{1}{2}X^2\mathrm{Li}_2(x)-\frac{1}{2}\pi^2\mathrm{Li}_2(x)-\frac{25}{3}Y\mathrm{Li}_2(y)-\frac{665}{72}\zeta_3+\frac{1}{2}Y\zeta_3-\\
& \quad -\frac{9}{2}X\zeta_3-\frac{23213}{5184}+\frac{31}{72}\pi^2+\frac{223}{1440}\pi^4-\frac{409}{432}X+\frac{23}{9}Y-\frac{5}{12}X^3Y-\frac{3}{4}Y^2\pi^2+
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{3}{8}X^2Y^2+X^2\pi^2-\frac{11}{12}XY\pi^2+\frac{19}{8}XY-\frac{8}{3}XY^2+\frac{23}{48}X\pi^2-\frac{1}{8}X^2Y-\\
&-\frac{53}{144}Y\pi^2+\frac{1}{12}XY^3-\frac{1}{8}Y^4-\frac{49}{12}Y^2+\frac{113}{72}Y^3\Bigg)+\\
&+i\pi\Bigg(3\frac{u}{t}Y+\frac{t}{s}\Bigg(6\mathrm{Li}_3(x)+6\mathrm{Li}_3(y)+6\mathrm{Li}_2(x)+\frac{1}{2}XY-Y\pi^2-\frac{1}{4}X^2+\frac{7}{4}X-\frac{7}{4}Y-\\
&\qquad-\frac{1}{12}\pi^2+\frac{11}{4}Y^2-Y^3+3XY^2\Bigg)+\\
&\quad+\Bigg(2\mathrm{Li}_3(x)-\mathrm{Li}_3(y)-X\mathrm{Li}_2(x)+\frac{28}{3}\mathrm{Li}_2(x)-Y\mathrm{Li}_2(x)-Y\mathrm{Li}_2(y)-4\zeta_3+\\
&\quad+\frac{19}{8}X-\frac{1673}{432}+\frac{11}{4}XY-\frac{1}{18}\pi^2-\frac{1}{12}Y^3+\frac{1}{4}X\pi^2-\frac{5}{8}X^2-X^2Y-\\
&\quad-\frac{5}{12}X^3-\frac{329}{72}Y+\frac{35}{8}Y^2-\frac{7}{12}Y\pi^2\Bigg)+\\
&+\frac{t^2}{s^2}\Bigg(7\mathrm{Li}_3(x)+7\mathrm{Li}_3(y)-X\mathrm{Li}_2(x)-Y\mathrm{Li}_2(y)-\frac{7}{6}Y\pi^2+\frac{3}{8}X^2-\frac{1}{6}X^3+\frac{3}{8}\pi^2+\\
&\quad+\frac{3}{8}Y^2-\frac{1}{3}X\pi^2+\frac{5}{2}XY^2-Y^3-\frac{3}{4}XY\Bigg)\Bigg)
\end{aligned}
\tag{A.58}
$$

$$
\begin{aligned}
B^{(2),[2]}_{u++++}=&\frac{t^2}{s^2}\Bigg(4\mathrm{Li}_4(x)+4\mathrm{Li}_4(y)+\frac{3}{2}\mathrm{Li}_4(z)-2X\mathrm{Li}_3(x)+\frac{1}{2}\mathrm{Li}_3(x)+\frac{1}{2}\mathrm{Li}_3(y)-4Y\mathrm{Li}_3(y)\\
&+2X\mathrm{Li}_3(y)-\frac{1}{2}X\mathrm{Li}_2(x)-\frac{1}{4}\pi^2\mathrm{Li}_2(x)+\frac{1}{2}XY\mathrm{Li}_2(x)+\frac{3}{4}X^2\mathrm{Li}_2(x)-\frac{1}{2}Y\mathrm{Li}_2(y)+\\
&+\frac{5}{4}Y^2\mathrm{Li}_2(y)-\frac{37}{144}\pi^4+\frac{127}{36}\pi^2+\frac{1}{12}XY^3+\frac{1}{4}X^3Y-\frac{1}{24}Y^2\pi^2+\frac{1}{2}X^2Y^2-\\
&-\frac{1}{24}X^2\pi^2-\frac{1}{4}XY\pi^2-\frac{127}{18}XY+\frac{11}{12}XY^2-\frac{7}{18}X\pi^2-\frac{1}{4}X^2Y-\frac{7}{9}Y\pi^2+\\
&+\frac{1}{16}Y^4-\frac{11}{36}X^3+\frac{1}{48}X^4+\frac{127}{36}X^2+\frac{127}{36}Y^2-\frac{25}{36}Y^3\Bigg)+\\
&+\frac{t}{s}\Bigg(\frac{5}{2}\mathrm{Li}_3(x)+\frac{5}{2}\mathrm{Li}_3(y)-\frac{5}{2}X\mathrm{Li}_2(x)-\frac{5}{2}Y\mathrm{Li}_2(y)+\frac{3}{2}\zeta_3-\frac{1}{3}Y^3-X^2Y+\frac{53}{18}\pi^2+\\
&+\frac{21}{4}Y^2+\frac{17}{12}X^2-\frac{1}{4}XY^2-\frac{2}{3}Y\pi^2-\frac{7}{12}X\pi^2-\frac{1}{12}X^3+\frac{32}{9}X-\\
&-\frac{20}{3}XY-\frac{32}{9}Y\Bigg)+2\frac{u}{t}Y^2+\\
&+\Bigg(-\frac{9}{2}\mathrm{Li}_4(z)-3X\mathrm{Li}_3(x)+\frac{7}{3}\mathrm{Li}_3(x)+2Y\mathrm{Li}_3(x)-4X\mathrm{Li}_3(y)+Y\mathrm{Li}_3(y)+\frac{8}{3}\mathrm{Li}_3(y)+\\
&+\frac{3}{4}X^2\mathrm{Li}_2(x)-\frac{7}{3}X\mathrm{Li}_2(x)+\frac{11}{4}\pi^2\mathrm{Li}_2(x)-\frac{5}{2}XY\mathrm{Li}_2(x)-\frac{8}{3}Y\mathrm{Li}_2(y)+\\
&+\frac{5}{4}Y^2\mathrm{Li}_2(y)-XY\mathrm{Li}_2(y)-2Y\zeta_3+2X\zeta_3+\frac{527}{72}\zeta_3+\frac{25}{18}X+\frac{325}{48}\pi^2-\frac{661}{1440}\pi^4+\\
&+\frac{46}{9}Y-\frac{3}{4}X^2\pi^2+\frac{29}{12}Y^2\pi^2-\frac{30659}{1296}+\frac{23}{12}XY\pi^2+\frac{1}{3}X^3Y-\frac{337}{72}XY-\frac{5}{2}XY^2-\\
&-\frac{2}{3}X\pi^2-\frac{1}{6}X^2Y+\frac{187}{36}Y\pi^2+\frac{2}{3}XY^3-\frac{19}{8}X^2Y^2-\frac{3}{16}Y^4-\frac{7}{18}X^3+\frac{1}{48}X^4+\\
&+\frac{125}{72}X^2-\frac{5}{18}Y^2+\frac{13}{36}Y^3\Bigg)+\\
&+i\pi\Bigg(\frac{t}{s}\Bigg(-2X+2Y\Bigg)+4\frac{u}{t}Y+
\end{aligned}
$$

$$+\left(-\mathrm{Li}_3(x)-3\mathrm{Li}_3(y)+2Y\mathrm{Li}_2(x)+\frac{1}{3}\mathrm{Li}_2(x)+6Y\mathrm{Li}_2(y)-\frac{9}{2}XY-\frac{39}{8}X+\right.$$
$$+\frac{11}{12}Y\pi^2+X^2Y+\frac{3}{4}X^2-\frac{49}{6}+\frac{1}{4}X\pi^2-Y^2+\frac{5}{2}XY^2+\frac{5}{12}X^3-\frac{13}{12}Y^3-$$
$$\left.-\frac{509}{72}Y+\frac{17}{18}\pi^2\right)$$
$$+\frac{t^2}{s^2}\left(-2\mathrm{Li}_3(x)-2\mathrm{Li}_3(y)+2X\mathrm{Li}_2(x)+2Y\mathrm{Li}_2(y)+\frac{1}{6}X^3+\frac{1}{2}Y\pi^2+\right.$$
$$\left.\left.+\frac{1}{2}XY^2+\frac{1}{6}Y^3+\frac{1}{2}X\pi^2+\frac{1}{2}X^2Y\right)\right) \quad (A.59)$$

$$C^{(2),[2]}_{u++++}=\frac{t^2}{s^2}\left(2\mathrm{Li}_4(x)+2\mathrm{Li}_4(y)+\frac{5}{2}\mathrm{Li}_4(z)-Y\mathrm{Li}_3(x)+\mathrm{Li}_3(x)+\mathrm{Li}_3(y)-3Y\mathrm{Li}_3(y)+\right.$$
$$+2X\mathrm{Li}_3(y)-\frac{5}{12}\pi^2\mathrm{Li}_2(x)-X\mathrm{Li}_2(x)+\frac{1}{4}X^2\mathrm{Li}_2(x)+\frac{3}{4}Y^2\mathrm{Li}_2(y)-\frac{1}{2}XY\mathrm{Li}_2(y)-$$
$$-Y\mathrm{Li}_2(y)-\frac{59}{720}\pi^4+2\pi^2-\frac{5}{4}X^2Y+\frac{5}{24}X\pi^2+\frac{5}{12}X^3Y-\frac{1}{12}XY^3-\frac{1}{4}X^2Y^2-$$
$$-\frac{1}{4}XY\pi^2-\frac{1}{3}Y\pi^2-4XY+\frac{3}{8}XY^2+\frac{1}{16}Y^4+\frac{3}{8}X^3-\frac{1}{16}X^4+2X^2+$$
$$\left.+2Y^2-\frac{1}{6}Y^3\right)+$$
$$+\left(-2\mathrm{Li}_4(x)+\mathrm{Li}_4(y)-\frac{7}{2}\mathrm{Li}_4(z)+2Y\mathrm{Li}_3(x)-X\mathrm{Li}_3(x)-\frac{1}{2}\mathrm{Li}_3(x)-3X\mathrm{Li}_3(y)-\right.$$
$$-\frac{1}{2}\mathrm{Li}_3(y)+Y\mathrm{Li}_3(y)+\frac{19}{12}\pi^2\mathrm{Li}_2(x)+\frac{1}{2}X\mathrm{Li}_2(x)+\frac{1}{4}X^2\mathrm{Li}_2(x)-3XY\mathrm{Li}_2(x)+$$
$$+\frac{1}{2}Y\mathrm{Li}_2(y)+\frac{1}{4}Y^2\mathrm{Li}_2(y)-\frac{3}{2}XY\mathrm{Li}_2(y)+4X\zeta_3+\frac{25}{4}\zeta_3-Y\zeta_3-3Y-\frac{163}{720}\pi^4-$$
$$-\frac{511}{64}+\frac{97}{48}\pi^2+\frac{1}{6}X^2\pi^2+\frac{1}{12}X^3Y+\frac{5}{12}Y^2\pi^2+\frac{5}{4}XY\pi^2-\frac{45}{16}X-\frac{15}{4}XY-$$
$$-\frac{3}{8}XY^2+\frac{5}{24}X\pi^2-\frac{1}{4}X^2Y+\frac{19}{12}Y\pi^2+\frac{7}{12}XY^3-\frac{11}{4}X^2Y^2-\frac{5}{48}Y^4+$$
$$\left.+\frac{5}{24}X^3-\frac{1}{48}X^4+\frac{5}{8}X^2\right)+$$
$$+\frac{t}{s}\left(\frac{1}{2}\mathrm{Li}_3(x)+\frac{1}{2}\mathrm{Li}_3(y)-\frac{1}{2}X\mathrm{Li}_2(x)-\frac{1}{2}Y\mathrm{Li}_2(y)+\frac{5}{2}\zeta_3+\frac{3}{2}\pi^2-\frac{1}{12}X\pi^2-\frac{15}{4}XY+\right.$$
$$\left.+3X-\frac{1}{6}X^3-\frac{1}{6}Y^3+2Y^2-3Y+\frac{7}{4}X^2-\frac{1}{4}Y\pi^2\right)+\frac{1}{2}\frac{u}{t}Y^2+$$
$$+i\pi\left(\frac{t^2}{s^2}\left(-\mathrm{Li}_3(x)-\mathrm{Li}_3(y)+X\mathrm{Li}_2(x)+Y\mathrm{Li}_2(y)+\frac{3}{8}\pi^2-\frac{3}{4}XY+\frac{1}{6}Y^3+\frac{1}{2}X^2Y+\right.\right.$$
$$\left.+\frac{1}{6}X\pi^2+\frac{3}{8}X^2+\frac{1}{3}Y\pi^2+\frac{3}{8}Y^2\right)+$$
$$+\frac{u}{t}Y+\frac{t}{s}\left(\frac{1}{2}XY+\frac{1}{4}Y-\frac{1}{12}\pi^2-\frac{1}{4}X-\frac{1}{4}Y^2-\frac{1}{4}X^2\right)+$$
$$+\left(\mathrm{Li}_3(x)-2\mathrm{Li}_3(y)-X\mathrm{Li}_2(x)+Y\mathrm{Li}_2(x)+3Y\mathrm{Li}_2(y)+3\zeta_3+\frac{3}{2}XY^2+\frac{1}{24}\pi^2-\right.$$
$$\left.\left.-\frac{5}{8}Y^2+\frac{1}{8}X^2+\frac{1}{2}X\pi^2-\frac{15}{4}Y-\frac{7}{4}XY-\frac{5}{2}X-\frac{93}{16}-\frac{1}{6}Y^3\right)\right) \quad (A.60)$$

$$D^{(2),[2]}_{u++++}=\left(-\frac{1}{3}\mathrm{Li}_3(y)+\frac{1}{3}Y\mathrm{Li}_2(y)+\frac{49}{36}\zeta_3-\frac{1}{24}X\pi^2+\frac{25}{108}X-\frac{5}{18}Y^3-\frac{25}{72}\pi^2+\frac{1}{6}XY^2+\right.$$

$$
\begin{aligned}
& + \frac{455}{108} - \frac{11}{72}Y\pi^2 - \frac{265}{108}Y + \frac{37}{36}Y^2\Bigg) + \\
& + i\pi\left(-\frac{1}{3}\mathrm{Li}_2(x) + \frac{149}{108} - \frac{1}{2}Y^2 + \frac{11}{18}Y - \frac{1}{36}\pi^2\right) \qquad \text{(A.61)}
\end{aligned}
$$

$$
\begin{aligned}
E^{(2),[2]}_{u++++} = & \frac{t}{s}\left(-\frac{1}{2}Y^2 + \frac{8}{9}Y - \frac{5}{18}\pi^2 - \frac{1}{6}X^2 - \frac{8}{9}X + \frac{2}{3}XY\right) + \\
& + \frac{t^2}{s^2}\left(\frac{1}{9}Y\pi^2 - \frac{4}{9}\pi^2 - \frac{4}{9}X^2 + \frac{1}{9}Y^3 + \frac{8}{9}XY - \frac{4}{9}Y^2 + \frac{1}{18}X^3 + \frac{1}{18}X\pi^2 - \frac{1}{6}XY^2\right) + \\
& + \Bigg(-\frac{1}{3}\mathrm{Li}_3(x) - \frac{2}{3}\mathrm{Li}_3(y) + \frac{1}{3}X\mathrm{Li}_2(x) + \frac{2}{3}Y\mathrm{Li}_2(y) + \frac{47}{36}\zeta_3 + \frac{1}{18}X^3 - \frac{1}{9}Y^3 + \frac{1}{18}Y\pi^2 + \\
& \quad + \frac{1}{6}X^2Y + \frac{7}{18}XY - \frac{1}{18}Y^2 + \frac{685}{162} - \frac{19}{36}X^2 + \frac{1}{36}X - \frac{13}{24}\pi^2 + \frac{1}{6}X\pi^2 - \frac{55}{36}Y\Bigg) + \\
& + i\pi\left(-\frac{1}{3}\mathrm{Li}_2(x) - \frac{1}{2}Y^2 + \frac{11}{18}Y + \frac{11}{12} + \frac{1}{18}\pi^2\right) \qquad \text{(A.62)}
\end{aligned}
$$

$$
F^{(2),[2]}_{u++++} = \left(-\frac{1}{9}Y^2 - \frac{25}{81} + \frac{10}{27}Y\right) \qquad \text{(A.63)}
$$

A.6 $q(p_2,+) + Q(p_1,-) \to Q(p_3,-) + q(p_4,+)$

$$
\begin{aligned}
A^{(2),[1]}_{u+--+} = & \frac{t}{s}\left(-3XY + \frac{3}{2}\pi^2 + \frac{3}{2}Y^2 + \frac{3}{2}X^2\right) + \\
& + \frac{u^2}{t^2}\Bigg(-3\mathrm{Li}_4(x) - 3\mathrm{Li}_4(y) - 3\mathrm{Li}_4(z) + 3Y\mathrm{Li}_3(y) - 3X\mathrm{Li}_3(y) + \frac{1}{2}\pi^2\mathrm{Li}_2(x) + 3X\zeta_3 - \\
& \quad - 3Y\zeta_3 + \frac{1}{2}XY^3 + \frac{1}{30}\pi^4 - \frac{3}{4}X^2Y^2 + \frac{1}{4}Y^2\pi^2 + XY\pi^2\Bigg) + \\
& + \frac{u}{t}\Bigg(-3\mathrm{Li}_4(x) - 3\mathrm{Li}_4(y) - 3\mathrm{Li}_4(z) - 3\mathrm{Li}_3(x) - 3X\mathrm{Li}_3(y) + 3Y\mathrm{Li}_3(y) - 3\mathrm{Li}_3(y) + \\
& \quad + 3X\mathrm{Li}_2(x) + \frac{1}{2}\pi^2\mathrm{Li}_2(x) + 3Y\mathrm{Li}_2(y) - 3Y\zeta_3 + 3X\zeta_3 + \frac{3}{2}X^2Y - \frac{3}{4}X^2Y^2 + \\
& \quad + \frac{1}{2}XY^3 - \frac{1}{2}\pi^2 + \frac{1}{2}X\pi^2 + \frac{1}{2}Y^3 + \frac{1}{30}\pi^4 + \frac{1}{4}Y^2\pi^2 + Y\pi^2 + XY\pi^2\Bigg) + \\
& + \Bigg(\mathrm{Li}_4(z) + X\mathrm{Li}_3(x) + \frac{11}{6}\mathrm{Li}_3(x) - Y\mathrm{Li}_3(x) + X\mathrm{Li}_3(y) - Y\mathrm{Li}_3(y) + \frac{11}{6}\mathrm{Li}_3(y) - \\
& \quad - \frac{1}{2}X^2\mathrm{Li}_2(x) - \frac{11}{6}X\mathrm{Li}_2(x) - \frac{1}{6}\pi^2\mathrm{Li}_2(x) + 2XY\mathrm{Li}_2(x) - \frac{11}{6}Y\mathrm{Li}_2(y) + \\
& \quad + \frac{1}{2}Y^2\mathrm{Li}_2(y) + XY\mathrm{Li}_2(y) + \frac{3}{2}Y\zeta_3 + 2X\zeta_3 + \frac{65}{72}\zeta_3 - \frac{95}{48}X + \frac{7}{2}\pi^2 + \frac{113}{1440}\pi^4 - \\
& \quad - \frac{23}{9}Y + \frac{1}{12}X^2\pi^2 + \frac{1}{4}Y^2\pi^2 + \frac{23213}{5184} - \frac{2}{3}XY\pi^2 - \frac{1}{2}X^3Y - \frac{125}{18}XY - \\
& \quad - \frac{5}{12}XY^2 - \frac{31}{18}X\pi^2 + \frac{4}{3}X^2Y - \frac{181}{144}Y\pi^2 - \frac{1}{2}XY^3 + \frac{9}{4}X^2Y^2 + \frac{1}{6}Y^4 - \\
& \quad - \frac{49}{36}X^3 + \frac{1}{12}X^4 + \frac{56}{9}X^2 + \frac{49}{12}Y^2 - \frac{7}{9}Y^3\Bigg) + \\
& + i\pi\left(\frac{7}{2}\zeta_3 + \frac{409}{432} - \frac{11}{48}\pi^2\right) \qquad \text{(A.64)}
\end{aligned}
$$

$$
B^{(2),[1]}_{u+--+} = \frac{u^2}{t^2}\Bigg(4\mathrm{Li}_4(x) + 2\mathrm{Li}_4(y) + 4\mathrm{Li}_4(z) + 2Y\mathrm{Li}_3(x) - 2Y\mathrm{Li}_3(y) + 5X\mathrm{Li}_3(y) - \mathrm{Li}_3(y) -
$$

$$
\begin{aligned}
&- \frac{3}{2}XY\mathrm{Li}_2(x) - \frac{1}{3}\pi^2\mathrm{Li}_2(x) - Y^2\mathrm{Li}_2(y) + Y\mathrm{Li}_2(y) - \frac{1}{2}XY\mathrm{Li}_2(y) - 5X\zeta_3 + \zeta_3 + \\
&+ 2Y\zeta_3 - \frac{29}{36}Y^3 + \frac{73}{18}Y^2 + \frac{1}{8}XY^2 + \frac{1}{2}X^2Y^2 - \frac{5}{6}XY^3 - \frac{19}{12}XY\pi^2 - \frac{19}{24}Y^2\pi^2 - \\
&- \frac{53}{36}Y\pi^2 - \frac{1}{45}\pi^4\Bigg) + \\
&+\frac{t}{s}\left(-\frac{7}{2}Y^2 - \frac{7}{2}X^2 + 7XY - \frac{7}{2}\pi^2\right) + \\
&+\frac{u}{t}\Bigg(2\mathrm{Li}_4(x) - 2\mathrm{Li}_4(y) + 2\mathrm{Li}_4(z) + 4Y\mathrm{Li}_3(x) + 6\mathrm{Li}_3(x) + 4\mathrm{Li}_3(y) + 4X\mathrm{Li}_3(y) + \\
&\quad + 2Y\mathrm{Li}_3(y) - 3XY\mathrm{Li}_2(x) + \frac{1}{3}\pi^2\mathrm{Li}_2(x) - 6X\mathrm{Li}_2(x) - 2Y^2\mathrm{Li}_2(y) - 4Y\mathrm{Li}_2(y) - \\
&\quad - XY\mathrm{Li}_2(y) - 2\zeta_3 - 2Y\zeta_3 - 4X\zeta_3 - \frac{41}{36}\pi^2 + \frac{1}{45}\pi^4 + \frac{64}{9}Y - 3X^2Y - \frac{205}{36}Y\pi^2 - \\
&\quad - \frac{1}{2}X^2Y^2 - \frac{13}{12}Y^2\pi^2 - \frac{7}{6}XY\pi^2 - \frac{2}{3}XY^3 + \frac{1}{4}XY - \frac{5}{6}X\pi^2 + \frac{65}{18}Y^2 - \frac{19}{9}Y^3\Bigg) + \\
&+\Bigg(-3\mathrm{Li}_4(z) + 3Y\mathrm{Li}_3(x) - \frac{11}{3}\mathrm{Li}_3(x) - 3X\mathrm{Li}_3(x) + 3Y\mathrm{Li}_3(y) - \frac{22}{3}\mathrm{Li}_3(y) - 3X\mathrm{Li}_3(y) + \\
&\quad + \frac{11}{3}X\mathrm{Li}_2(x) + \frac{1}{2}\pi^2\mathrm{Li}_2(x) + \frac{3}{2}X^2\mathrm{Li}_2(x) - 7XY\mathrm{Li}_2(x) + \frac{22}{3}Y\mathrm{Li}_2(y) - \\
&\quad - \frac{3}{2}Y^2\mathrm{Li}_2(y) - 4XY\mathrm{Li}_2(y) - 3Y\zeta_3 - \frac{179}{72}\zeta_3 + \frac{7}{2}X\zeta_3 + \frac{30659}{1296} - \frac{1469}{144}\pi^2 - \\
&\quad - \frac{49}{54}X - \frac{26}{3}Y + \frac{3}{2}X^3Y - \frac{521}{1440}\pi^4 - \frac{15}{2}X^2Y^2 + \frac{7}{3}XY\pi^2 - \frac{5}{4}Y^2\pi^2 + \\
&\quad + \frac{98}{9}XY + \frac{8}{3}XY^2 + \frac{535}{144}X\pi^2 - \frac{25}{6}X^2Y - \frac{7}{6}Y\pi^2 + XY^3 - \frac{1}{8}Y^4 + \frac{125}{36}X^3 \\
&\quad - \frac{1}{4}X^4 - \frac{869}{72}X^2 - \frac{3}{2}Y^2\Bigg) + \\
&+i\pi\Bigg(\Bigg(-\frac{11}{3}\mathrm{Li}_2(x) + \frac{1}{2}\zeta_3 + \frac{1}{2}X^2Y + \frac{1}{6}Y\pi^2 + Y^3 + \frac{275}{54} + \frac{3}{4}X^2 - XY^2 - \frac{9}{4}X + \\
&\qquad + \frac{55}{144}\pi^2 + \frac{2}{3}X\pi^2 - 3XY - Y^2 + \frac{71}{9}Y\Bigg) + \\
&\qquad + \frac{u}{t}\Bigg(4\mathrm{Li}_3(x) + 6\mathrm{Li}_3(y) - 2X\mathrm{Li}_2(x) - 2\mathrm{Li}_2(x) - 2Y\mathrm{Li}_2(y) - 6\zeta_3 - \frac{5}{6}Y\pi^2 - \\
&\qquad\quad - \frac{2}{3}Y^2 - 2XY - \frac{1}{12}\pi^2 + \frac{64}{9} + \frac{1}{3}Y^3 + \frac{1}{4}X + \frac{401}{36}Y\Bigg) + \\
&\qquad + \frac{u^2}{t^2}\Bigg(2\mathrm{Li}_3(x) + 3\mathrm{Li}_3(y) - \mathrm{Li}_2(x) - X\mathrm{Li}_2(x) - Y\mathrm{Li}_2(y) - 3\zeta_3 - \frac{5}{12}Y\pi^2 + \\
&\qquad\quad + \frac{73}{9}Y - \frac{23}{24}Y^2 + \frac{1}{6}Y^3 - \frac{3}{4}XY\Bigg)\Bigg) \qquad \text{(A.65)}
\end{aligned}
$$

$$
\begin{aligned}
C^{(2),[1]}_{u+--+} = \frac{u}{t}\Bigg(&12\mathrm{Li}_4(x) + 12\mathrm{Li}_4(z) - 12Y\mathrm{Li}_3(x) + 2\mathrm{Li}_3(y) + 6X\mathrm{Li}_3(y) - 8Y\mathrm{Li}_3(y) - \\
&- 4\pi^2\mathrm{Li}_2(x) + 9XY\mathrm{Li}_2(x) - 2Y\mathrm{Li}_2(y) + 2Y^2\mathrm{Li}_2(y) + 3XY\mathrm{Li}_2(y) + 8Y\zeta_3 - \\
&- 2\zeta_3 - 6X\zeta_3 + 6Y + 2XY^2 - \frac{1}{2}X\pi^2 - \frac{55}{12}Y\pi^2 + \frac{11}{12}\pi^2 + 6X^2Y^2 - \frac{7}{12}Y^2\pi^2 - \\
&- \frac{5}{2}XY\pi^2 - XY^3 - \frac{3}{4}XY + \frac{1}{3}Y^4 + \frac{3}{2}Y^2 - \frac{1}{3}Y^3\Bigg) +
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{t}{s}\left(-3XY+\frac{3}{2}\pi^2+\frac{3}{2}Y^2+\frac{3}{2}X^2\right)\\
&+\Big(-8\mathrm{Li}_4(y)+4\mathrm{Li}_4(z)-4Y\mathrm{Li}_3(x)+4X\mathrm{Li}_3(x)+4X\mathrm{Li}_3(y)+4Y\mathrm{Li}_3(y)-\\
&\quad-\frac{10}{3}\pi^2\mathrm{Li}_2(x)-2X^2\mathrm{Li}_2(x)+10XY\mathrm{Li}_2(x)-2Y^2\mathrm{Li}_2(y)+6XY\mathrm{Li}_2(y)+4Y\zeta_3-\\
&\quad-\frac{15}{4}\zeta_3-7X\zeta_3-\frac{253}{48}\pi^2+\frac{189}{16}X+\frac{44}{45}\pi^4+\frac{511}{64}-\frac{3}{2}X^3Y-\frac{4}{3}XY^3+\frac{19}{2}X^2Y^2-\\
&\quad-\frac{7}{12}X^2\pi^2-\frac{19}{6}XY\pi^2-\frac{7}{6}Y^2\pi^2+5XY-2X\pi^2+\frac{9}{2}X^2Y-\frac{9}{2}Y\pi^2+\frac{1}{6}Y^4-\\
&\quad-\frac{9}{4}X^3+\frac{5}{24}X^4+\frac{7}{8}X^2+\frac{3}{2}Y^2\Big)+\\
&+\frac{u^2}{t^2}\Big(6\mathrm{Li}_4(x)+6\mathrm{Li}_4(z)-6Y\mathrm{Li}_3(x)-4Y\mathrm{Li}_3(y)+3\mathrm{Li}_3(y)+3X\mathrm{Li}_3(y)+\\
&\qquad+\frac{9}{2}XY\mathrm{Li}_2(x)-2\pi^2\mathrm{Li}_2(x)+\frac{3}{2}XY\mathrm{Li}_2(y)+Y^2\mathrm{Li}_2(y)-3Y\mathrm{Li}_2(y)-3X\zeta_3-\\
&\qquad-3\zeta_3+4Y\zeta_3-\frac{13}{12}Y\pi^2+\frac{3}{2}Y^2-\frac{1}{3}Y^3+3X^2Y^2-\frac{5}{4}XY\pi^2-\frac{1}{2}XY^3+\\
&\qquad+\frac{1}{6}Y^4-\frac{3}{8}XY^2-\frac{7}{24}Y^2\pi^2\Big)+\\
&+i\pi\Bigg(\frac{u^2}{t^2}\Big(-6\mathrm{Li}_3(x)-\mathrm{Li}_3(y)+3\mathrm{Li}_2(x)+3X\mathrm{Li}_2(x)-Y\mathrm{Li}_2(x)-2Y\mathrm{Li}_2(y)+\zeta_3+\\
&\qquad\quad+\frac{1}{8}Y^2+3Y-XY^2+\frac{5}{12}Y\pi^2+\frac{9}{4}XY+\frac{1}{6}Y^3\Big)+\\
&\qquad+\frac{u}{t}\Big(-12\mathrm{Li}_3(x)-2\mathrm{Li}_3(y)+2\mathrm{Li}_2(x)+6X\mathrm{Li}_2(x)-2Y\mathrm{Li}_2(x)-4Y\mathrm{Li}_2(y)+2\zeta_3+\\
&\qquad\quad+\frac{1}{3}Y^3-2XY^2+6+\frac{9}{4}Y+\frac{5}{6}Y\pi^2-\frac{1}{12}\pi^2-\frac{3}{4}X+6XY+2Y^2\Big)+\\
&\qquad+\Big(8\mathrm{Li}_3(y)-2Y\mathrm{Li}_2(x)-10Y\mathrm{Li}_2(y)-3\zeta_3+8Y+\frac{189}{16}-3XY^2-\frac{9}{4}X^2-\\
&\qquad\quad-\frac{3}{2}X^2Y+\frac{27}{4}X-\frac{7}{6}Y\pi^2-2X\pi^2-2\pi^2+9XY\Big)\Bigg)
\end{aligned}
\tag{A.66}
$$

$$
\begin{aligned}
D^{(2),[1]}_{u+--+}=&\Big(-\frac{1}{3}\mathrm{Li}_3(x)-\frac{1}{3}\mathrm{Li}_3(y)+\frac{1}{3}X\mathrm{Li}_2(x)+\frac{1}{3}Y\mathrm{Li}_2(y)-\frac{37}{36}\zeta_3-\frac{455}{108}+\frac{5}{18}Y^3+\\
&+\frac{265}{108}Y+\frac{1}{9}X^3+\frac{1}{6}X^2Y-\frac{11}{24}\pi^2+\frac{23}{72}Y\pi^2+\frac{2}{9}X\pi^2-\frac{37}{36}Y^2-\\
&-\frac{29}{36}X^2-\frac{1}{3}XY^2+\frac{11}{18}XY+\frac{11}{12}X\Big)+i\pi\left(-\frac{25}{108}+\frac{1}{24}\pi^2\right)
\end{aligned}
\tag{A.67}
$$

$$
\begin{aligned}
E^{(2),[1]}_{u+--+}=&\frac{u^2}{t^2}\left(\frac{2}{9}Y^3-\frac{8}{9}Y^2+\frac{2}{9}Y\pi^2\right)+\\
&+\Big(\frac{2}{3}\mathrm{Li}_3(x)+\frac{4}{3}\mathrm{Li}_3(y)-\frac{2}{3}X\mathrm{Li}_2(x)-\frac{4}{3}Y\mathrm{Li}_2(y)-\frac{59}{36}\zeta_3-\frac{223}{108}X-\frac{29}{72}X\pi^2+\frac{29}{12}Y-\\
&\quad-\frac{1}{3}X^2Y+\frac{1}{3}XY^2+\frac{29}{18}X^2-\frac{1}{3}Y\pi^2-\frac{2}{9}X^3-\frac{685}{162}-\frac{11}{9}XY+\frac{109}{72}\pi^2\Big)+\\
&+\frac{u}{t}\left(\frac{2}{9}\pi^2+\frac{4}{9}Y^3-\frac{7}{9}Y^2+\frac{4}{9}Y\pi^2-\frac{16}{9}Y\right)+
\end{aligned}
$$

$$+i\pi\left(\frac{u}{t}\left(-\frac{20}{9}Y-\frac{16}{9}+\frac{2}{3}Y^2\right)+\frac{u^2}{t^2}\left(-\frac{16}{9}Y+\frac{1}{3}Y^2\right)+\right.$$
$$\left.+\left(\frac{2}{3}\mathrm{Li}_2(x)-\frac{5}{72}\pi^2-\frac{11}{9}Y-\frac{223}{108}+Y^2\right)\right) \tag{A.68}$$

$$F^{(2),[1]}_{u+--+}=\left(\frac{1}{9}Y^2+\frac{25}{81}-\frac{10}{27}Y\right) \tag{A.69}$$

$$\begin{aligned}
A^{(2),[2]}_{u+--+} =\ & \frac{t}{s}\left(\frac{1}{2}X^2-XY+\frac{1}{2}\pi^2+\frac{1}{2}Y^2\right)+\\
&+\frac{u^2}{t^2}\left(-3\mathrm{Li}_4(x)+\frac{1}{2}\mathrm{Li}_4(y)-3\mathrm{Li}_4(z)-3X\mathrm{Li}_3(y)+Y\mathrm{Li}_3(y)+\frac{3}{2}\mathrm{Li}_3(y)+\frac{1}{6}\pi^2\mathrm{Li}_2(x)+\right.\\
&\quad+\frac{1}{4}Y^2\mathrm{Li}_2(y)-\frac{3}{2}Y\mathrm{Li}_2(y)-\frac{3}{2}\zeta_3+3X\zeta_3-Y\zeta_3+\frac{1}{3}Y^2\pi^2+\frac{13}{36}Y\pi^2+\frac{1}{9}Y^3-\\
&\quad\left.-\frac{1}{12}Y^4-\frac{3}{4}X^2Y^2+\frac{1}{2}XY^3+\frac{2}{3}XY\pi^2-\frac{37}{36}Y^2-\frac{1}{180}\pi^4\right)+\\
&+\frac{u}{t}\left(-3\mathrm{Li}_4(x)+4\mathrm{Li}_4(y)-3\mathrm{Li}_4(z)-3\mathrm{Li}_3(x)+\frac{3}{2}\mathrm{Li}_3(y)-3X\mathrm{Li}_3(y)-Y\mathrm{Li}_3(y)-\right.\\
&\quad-\frac{1}{6}\pi^2\mathrm{Li}_2(x)+3X\mathrm{Li}_2(x)+\frac{1}{2}Y^2\mathrm{Li}_2(y)-\frac{3}{2}Y\mathrm{Li}_2(y)+3X\zeta_3-\zeta_3+Y\zeta_3-\frac{32}{9}Y+\\
&\quad+\frac{7}{36}\pi^2-\frac{2}{45}\pi^4+\frac{3}{2}X^2Y+\frac{14}{9}Y\pi^2+\frac{1}{2}XY^3-\frac{3}{4}X^2Y^2+\frac{5}{12}Y^2\pi^2+\frac{1}{3}XY\pi^2+\\
&\quad\left.+\frac{3}{2}XY-\frac{1}{4}XY^2+\frac{1}{6}X\pi^2-\frac{1}{6}Y^4-\frac{83}{36}Y^2+\frac{5}{9}Y^3\right)+\\
&+\left(-4\mathrm{Li}_4(x)+\mathrm{Li}_4(y)+2\mathrm{Li}_4(z)+4X\mathrm{Li}_3(x)+\frac{3}{2}\mathrm{Li}_3(x)-Y\mathrm{Li}_3(x)+\frac{29}{6}\mathrm{Li}_3(y)-\right.\\
&\quad-Y\mathrm{Li}_3(y)+X\mathrm{Li}_3(y)-\frac{3}{2}X\mathrm{Li}_2(x)-\frac{2}{3}\pi^2\mathrm{Li}_2(x)-X^2\mathrm{Li}_2(x)-\frac{1}{2}Y^2\mathrm{Li}_2(y)-\\
&\quad-\frac{29}{6}Y\mathrm{Li}_2(y)-\frac{9}{2}X\zeta_3-\frac{305}{72}\zeta_3+\frac{1}{2}Y\zeta_3-\frac{409}{432}X+\frac{151}{72}\pi^2-\frac{23213}{5184}+\frac{9}{8}X^2\pi^2+\\
&\quad+\frac{95}{288}\pi^4-\frac{5}{8}Y^2\pi^2+\frac{23}{9}Y-\frac{7}{6}XY\pi^2-\frac{7}{12}X^3Y+\frac{1}{8}XY-\frac{11}{12}XY^2-\frac{1}{48}X\pi^2-\\
&\quad\left.-\frac{3}{4}X^2Y+\frac{49}{144}Y\pi^2-\frac{5}{12}XY^3+\frac{5}{8}X^2Y^2-\frac{1}{24}Y^4+\frac{3}{4}X^2-\frac{25}{12}Y^2+\frac{7}{9}Y^3\right)+\\
&+i\pi\left(\left(3\mathrm{Li}_3(x)-2X\mathrm{Li}_2(x)+\frac{10}{3}\mathrm{Li}_2(x)-Y\mathrm{Li}_2(x)-2Y\mathrm{Li}_2(y)-4\zeta_3-\frac{1673}{432}+\right.\right.\\
&\quad\quad+\frac{3}{2}XY-\frac{1}{2}XY^2-\frac{7}{12}X^3-\frac{1}{12}X\pi^2-\frac{1}{12}Y^3+\frac{29}{72}\pi^2-\frac{203}{72}Y+\frac{13}{8}X+\\
&\quad\quad\left.+2Y^2-X^2Y-\frac{3}{4}Y\pi^2\right)+\\
&\quad+\frac{u}{t}\left(-4\mathrm{Li}_3(y)-Y\mathrm{Li}_2(x)+\frac{9}{2}\mathrm{Li}_2(x)+4\zeta_3+\frac{1}{2}Y\pi^2-\frac{32}{9}-\frac{1}{3}Y^3-XY^2+\right.\\
&\quad\quad\left.+\frac{1}{3}Y^2-\frac{1}{12}\pi^2-\frac{89}{18}Y+\frac{3}{2}X+4XY\right)+\\
&\quad+\frac{u^2}{t^2}\left(-2\mathrm{Li}_3(y)-\frac{1}{2}Y\mathrm{Li}_2(x)+\frac{3}{2}\mathrm{Li}_2(x)+2\zeta_3+\frac{1}{4}Y\pi^2+\frac{3}{2}XY+\right.\\
&\quad\quad\left.\left.+\frac{1}{6}Y^2-\frac{1}{6}Y^3-\frac{1}{2}XY^2-\frac{37}{18}Y\right)\right)
\end{aligned} \tag{A.70}$$

$$
\begin{aligned}
B^{(2),[2]}_{u+--+} = & \Big(4\mathrm{Li}_4(x)+4\mathrm{Li}_4(y)-3\mathrm{Li}_4(z)+2Y\mathrm{Li}_3(x)-5X\mathrm{Li}_3(x)+\frac{1}{3}\mathrm{Li}_3(x)-2X\mathrm{Li}_3(y)+\\
&+\frac{2}{3}\mathrm{Li}_3(y)-3Y\mathrm{Li}_3(y)+\frac{5}{2}\pi^2\mathrm{Li}_2(x)-\frac{1}{3}X\mathrm{Li}_2(x)+\frac{3}{2}X^2\mathrm{Li}_2(x)-2XY\mathrm{Li}_2(x)-\\
&-\frac{2}{3}Y\mathrm{Li}_2(y)+\frac{5}{2}Y^2\mathrm{Li}_2(y)-XY\mathrm{Li}_2(y)+2X\zeta_3+\frac{419}{72}\zeta_3-2Y\zeta_3+\frac{26}{3}Y-\frac{1031}{1440}\pi^4-\\
&-\frac{30659}{1296}+\frac{377}{48}\pi^2-\frac{19}{24}X^2\pi^2+\frac{7}{12}X^3Y+\frac{19}{8}Y^2\pi^2+\frac{5}{3}XY\pi^2-\frac{13}{6}X-\frac{437}{72}XY-\\
&-\frac{4}{3}XY^2-\frac{17}{36}X\pi^2+\frac{7}{12}X^2Y+\frac{61}{12}Y\pi^2+\frac{3}{4}XY^3-\frac{15}{8}X^2Y^2-\frac{1}{8}Y^4-\frac{11}{18}X^3+\\
&+\frac{1}{24}X^4+\frac{277}{72}X^2-\frac{3}{2}Y^2\Big)+\\
&+\frac{u^2}{t^2}\Big(-2\mathrm{Li}_4(x)-2\mathrm{Li}_4(z)+2Y\mathrm{Li}_3(x)-\frac{5}{2}\mathrm{Li}_3(y)-X\mathrm{Li}_3(y)+Y\mathrm{Li}_3(y)+\pi^2\mathrm{Li}_2(x)-\\
&\quad-\frac{3}{2}XY\mathrm{Li}_2(x)-\frac{1}{2}XY\mathrm{Li}_2(y)+\frac{5}{2}Y\mathrm{Li}_2(y)-Y\zeta_3+X\zeta_3+\frac{5}{2}\zeta_3+\frac{35}{18}Y\pi^2+\\
&\quad+\frac{1}{6}XY^3+\frac{31}{36}Y^3-X^2Y^2+\frac{3}{4}XY\pi^2+\frac{11}{24}Y^2\pi^2-\frac{127}{36}Y^2+\frac{1}{8}XY^2-\frac{1}{24}Y^4\Big)+\\
&+\frac{u}{t}\Big(-4\mathrm{Li}_4(x)-4\mathrm{Li}_4(z)+4Y\mathrm{Li}_3(x)-4\mathrm{Li}_3(y)-2X\mathrm{Li}_3(y)+2Y\mathrm{Li}_3(y)-3XY\mathrm{Li}_2(x)+\\
&\quad+2\pi^2\mathrm{Li}_2(x)+4Y\mathrm{Li}_2(y)-XY\mathrm{Li}_2(y)+4\zeta_3-2Y\zeta_3+2X\zeta_3-\frac{59}{9}Y+\\
&\quad+\frac{17}{18}\pi^2-XY^2+\frac{1}{2}X\pi^2+\frac{227}{36}Y\pi^2-\frac{5}{4}XY-2X^2Y^2+\frac{11}{12}Y^2\pi^2+\frac{3}{2}XY\pi^2+\\
&\quad+\frac{1}{3}XY^3-\frac{1}{12}Y^4-\frac{65}{36}Y^2+\frac{11}{9}Y^3\Big)+\\
&+\frac{t}{s}\Big(\frac{1}{2}X^2-XY+\frac{1}{2}\pi^2+\frac{1}{2}Y^2\Big)+\\
&+i\pi\Big(\Big(-3\mathrm{Li}_3(x)-5\mathrm{Li}_3(y)+\frac{1}{3}\mathrm{Li}_2(x)+2X\mathrm{Li}_2(x)+2Y\mathrm{Li}_2(x)+8Y\mathrm{Li}_2(y)+\\
&\quad+\frac{3}{2}X^2Y+3XY^2+\frac{7}{12}X^3+\frac{17}{18}\pi^2+\frac{3}{4}X^2-\frac{653}{72}Y+\frac{3}{4}X\pi^2-\frac{11}{12}Y^3-\\
&\quad-\frac{9}{2}XY-Y^2-\frac{49}{6}+\frac{17}{12}Y\pi^2-\frac{31}{8}X\Big)+\\
&\quad+\frac{u}{t}\Big(4\mathrm{Li}_3(x)-2X\mathrm{Li}_2(x)-4\mathrm{Li}_2(x)+2Y\mathrm{Li}_2(x)+4Y\mathrm{Li}_2(y)-6XY-\frac{1}{3}Y^3-\\
&\qquad-\frac{5}{4}X-\frac{1}{6}Y\pi^2-\frac{59}{9}-\frac{7}{6}Y^2-\frac{241}{36}Y+2XY^2+\frac{1}{4}\pi^2\Big)+\\
&\quad+\frac{u^2}{t^2}\Big(2\mathrm{Li}_3(x)-X\mathrm{Li}_2(x)-\frac{5}{2}\mathrm{Li}_2(x)+Y\mathrm{Li}_2(x)+2Y\mathrm{Li}_2(y)-\frac{1}{12}Y\pi^2+\\
&\qquad+XY^2-\frac{9}{4}XY-\frac{127}{18}Y-\frac{1}{6}Y^3+\frac{13}{24}Y^2\Big)\Big)
\end{aligned}\tag{A.71}
$$

$$
\begin{aligned}
C^{(2),[2]}_{u+--+} = & \frac{u^2}{t^2}\Big(-2\mathrm{Li}_4(x)+\frac{1}{2}\mathrm{Li}_4(y)-2\mathrm{Li}_4(z)+2Y\mathrm{Li}_3(x)-\mathrm{Li}_3(y)-X\mathrm{Li}_3(y)+Y\mathrm{Li}_3(y)+\\
&+\frac{2}{3}\pi^2\mathrm{Li}_2(x)-\frac{3}{2}XY\mathrm{Li}_2(x)-\frac{1}{4}Y^2\mathrm{Li}_2(y)-\frac{1}{2}XY\mathrm{Li}_2(y)+Y\mathrm{Li}_2(y)+\zeta_3-Y\zeta_3+\\
&+X\zeta_3+\frac{1}{8}XY^2+\frac{1}{4}Y\pi^2-X^2Y^2+\frac{1}{24}Y^2\pi^2-\frac{1}{180}\pi^4+\frac{1}{6}XY^3+
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{5}{12}XY\pi^2-\frac{1}{24}Y^4-Y^2+\frac{1}{6}Y^3\Big)+\\
&+\frac{t}{s}\Big(XY-\frac{1}{2}\pi^2-\frac{1}{2}Y^2-\frac{1}{2}X^2\Big)\\
&+\Big(3\mathrm{Li}_4(y)-\mathrm{Li}_4(z)+Y\mathrm{Li}_3(x)-X\mathrm{Li}_3(x)-X\mathrm{Li}_3(y)-2Y\mathrm{Li}_3(y)+\frac{7}{6}\pi^2\mathrm{Li}_2(x)+\\
&\quad+\frac{1}{2}X^2\mathrm{Li}_2(x)-3XY\mathrm{Li}_2(x)+Y^2\mathrm{Li}_2(y)-2XY\mathrm{Li}_2(y)-Y\zeta_3+\frac{15}{4}\zeta_3+4X\zeta_3+\\
&\quad+\frac{97}{48}\pi^2-\frac{93}{16}X-\frac{37}{120}\pi^4-\frac{511}{64}+\frac{1}{2}X^3Y+\frac{1}{2}XY^3-3X^2Y^2+\frac{1}{6}X^2\pi^2+\\
&\quad+XY\pi^2+\frac{5}{12}Y^2\pi^2-3XY+\frac{1}{2}X\pi^2-\frac{3}{2}X^2Y+\frac{3}{2}Y\pi^2-\frac{1}{24}Y^4+\\
&\quad+\frac{3}{4}X^3-\frac{1}{12}X^4+\frac{3}{8}X^2-\frac{1}{2}Y^2\Big)+\\
&+\frac{u}{t}\Big(-4\mathrm{Li}_4(x)+\mathrm{Li}_4(y)-4\mathrm{Li}_4(z)+4Y\mathrm{Li}_3(x)-\frac{1}{2}\mathrm{Li}_3(y)-2X\mathrm{Li}_3(y)+2Y\mathrm{Li}_3(y)-\\
&\quad-3XY\mathrm{Li}_2(x)+\frac{4}{3}\pi^2\mathrm{Li}_2(x)-\frac{1}{2}Y^2\mathrm{Li}_2(y)+\frac{1}{2}Y\mathrm{Li}_2(y)-XY\mathrm{Li}_2(y)+\zeta_3-\\
&\quad-2Y\zeta_3+2X\zeta_3-3Y-\frac{5}{12}\pi^2-\frac{3}{4}XY^2+\frac{1}{6}X\pi^2+\frac{17}{12}Y\pi^2-\frac{1}{90}\pi^4-2X^2Y^2+\\
&\quad+\frac{1}{12}Y^2\pi^2+\frac{5}{6}XY\pi^2+\frac{1}{3}XY^3+\frac{1}{4}XY-\frac{1}{12}Y^4-Y^2+\frac{1}{6}Y^3\Big)+\\
&+i\pi\Big(\frac{u^2}{t^2}\Big(2\mathrm{Li}_3(x)-X\mathrm{Li}_2(x)+\frac{1}{2}Y\mathrm{Li}_2(x)-\mathrm{Li}_2(x)+Y\mathrm{Li}_2(y)+\frac{1}{2}XY^2-\frac{1}{6}Y\pi^2\\
&\qquad+\frac{1}{8}Y^2-2Y-\frac{3}{4}XY\Big)\\
&\quad+\frac{u}{t}\Big(4\mathrm{Li}_3(x)-2X\mathrm{Li}_2(x)+Y\mathrm{Li}_2(x)-\frac{1}{2}\mathrm{Li}_2(x)+2Y\mathrm{Li}_2(y)-\frac{7}{4}Y+XY^2+\\
&\qquad+\frac{1}{4}X-\frac{1}{3}Y\pi^2-2XY-3-\frac{1}{2}Y^2\Big)+\\
&\quad+\Big(-3\mathrm{Li}_3(y)+Y\mathrm{Li}_2(x)+4Y\mathrm{Li}_2(y)+3\zeta_3+\frac{1}{2}\pi^2+\frac{3}{2}XY^2-3XY+\frac{1}{3}Y\pi^2+\\
&\qquad+\frac{1}{2}X^2Y-\frac{9}{4}X-\frac{93}{16}+\frac{2}{3}X\pi^2+\frac{3}{4}X^2-4Y\Big)\Big)
\end{aligned}\tag{A.72}
$$

$$
\begin{aligned}
D^{(2),[2]}_{u+--+}=&\frac{u^2}{t^2}\Big(\frac{4}{9}Y^2-\frac{1}{9}Y^3-\frac{1}{9}Y\pi^2\Big)+\frac{u}{t}\Big(\frac{7}{18}Y^2-\frac{2}{9}Y^3-\frac{2}{9}Y\pi^2-\frac{1}{9}\pi^2+\frac{8}{9}Y\Big)+\\
&+\Big(-\frac{1}{3}\mathrm{Li}_3(y)+\frac{1}{3}Y\mathrm{Li}_2(y)+\frac{49}{36}\zeta_3-\frac{1}{24}X\pi^2+\frac{25}{108}X-\frac{5}{18}Y^3-\frac{25}{72}\pi^2+\frac{1}{6}XY^2+\\
&\quad+\frac{455}{108}-\frac{11}{72}Y\pi^2-\frac{265}{108}Y+\frac{37}{36}Y^2\Big)+\\
&+i\pi\Big(\Big(-\frac{1}{3}\mathrm{Li}_2(x)+\frac{149}{108}-\frac{1}{2}Y^2+\frac{11}{18}Y-\frac{1}{36}\pi^2\Big)+\\
&\quad+\frac{u}{t}\Big(-\frac{1}{3}Y^2+\frac{8}{9}+\frac{10}{9}Y\Big)+\frac{u^2}{t^2}\Big(\frac{8}{9}Y-\frac{1}{6}Y^2\Big)\Big)
\end{aligned}\tag{A.73}
$$

$$
E^{(2),[2]}_{u+--+}=\Big(-\frac{1}{3}\mathrm{Li}_3(x)-\frac{2}{3}\mathrm{Li}_3(y)+\frac{1}{3}X\mathrm{Li}_2(x)+\frac{2}{3}Y\mathrm{Li}_2(y)+\frac{47}{36}\zeta_3-\frac{29}{36}X^2+\frac{1}{6}X^2Y+
$$

$$+\frac{685}{162}+\frac{1}{6}Y\pi^2-\frac{29}{12}Y+\frac{1}{9}X^3+\frac{11}{18}XY+\frac{2}{9}X\pi^2-\frac{1}{6}XY^2-\frac{17}{24}\pi^2+\frac{11}{12}X\Bigg)+$$
$$+\frac{u}{t}\left(\frac{7}{18}Y^2-\frac{2}{9}Y^3-\frac{2}{9}Y\pi^2-\frac{1}{9}\pi^2+\frac{8}{9}Y\right)+$$
$$+\frac{u^2}{t^2}\left(\frac{4}{9}Y^2-\frac{1}{9}Y^3-\frac{1}{9}Y\pi^2\right)+$$
$$+i\pi\left(\frac{u^2}{t^2}\left(\frac{8}{9}Y-\frac{1}{6}Y^2\right)+\frac{u}{t}\left(-\frac{1}{3}Y^2+\frac{8}{9}+\frac{10}{9}Y\right)+\right.$$
$$\left.+\left(-\frac{1}{3}\mathrm{Li}_2(x)-\frac{1}{2}Y^2+\frac{11}{18}Y+\frac{11}{12}+\frac{1}{18}\pi^2\right)\right) \tag{A.74}$$
$$F^{(2),[2]}_{u+--+} = \left(-\frac{1}{9}Y^2-\frac{25}{81}+\frac{10}{27}Y\right) \tag{A.75}$$

A.7 $q(p_2,+)+Q(p_1,+)\to q(p_3,+)+Q(p_4,+)$

The coefficients for this amplitude are obtained by exchanging t and u in the coefficients for $q(p_2,+)+Q(p_1,+)\to Q(p_3,+)+q(p_4,+)$,

$$A^{(2),[1]}_{t'++++} = A^{(2),[1]}_{u++++}(t\leftrightarrow u) \tag{A.76}$$
$$A^{(2),[2]}_{t'++++} = A^{(2),[2]}_{u++++}(t\leftrightarrow u) \tag{A.77}$$

etc.

A.8 $q(p_2,+)+Q(p_1,-)\to q(p_3,+)+Q(p_4,-)$

The coefficients for this amplitude are obtained by exchanging t and u in the coefficients for $q(p_2,+)+Q(p_1,-)\to Q(p_3,-)+q(p_4,+)$,

$$A^{(2),[1]}_{t'+-+-} = A^{(2),[1]}_{u+--+}(t\leftrightarrow u) \tag{A.78}$$
$$A^{(2),[2]}_{t'+-+-} = A^{(2),[2]}_{u+--+}(t\leftrightarrow u) \tag{A.79}$$

etc.

B. Finite one-loop contributions

In this appendix we list the coefficients $A,\dots,C$ for the finite one-loop amplitudes defined in section 4.3 for the four quark scattering processes of eqs. (4.13)–(4.15).

B.1 $q(p_2,+)+\bar{q}(p_1,-)\to Q(p_3,+)+\bar{Q}(p_4,-)$

$$A^{(1),[1]}_{s+-+-} = \frac{1}{2}\frac{t}{u}X+\frac{1}{4}\frac{t^2}{u^2}X^2+\left(\frac{1}{4}X^2-X+\frac{13}{18}\right)+$$
$$+i\pi\left(\left(\frac{5}{6}+\frac{1}{2}X\right)+\frac{1}{2}\frac{t^2}{u^2}X+\frac{1}{2}\frac{t}{u}\right) \tag{B.1}$$
$$B^{(1),[1]}_{s+-+-} = -\frac{t}{u}X-\frac{1}{2}\frac{t^2}{u^2}X^2+\left(2X+Y^2-\frac{1}{2}X^2-3Y+4\right)+$$

$$+i\pi\Big(-\frac{t^2}{u^2}X-\frac{t}{u}+\Big(-X+2Y-1\Big)\Big) \tag{B.2}$$

$$C^{(1),[1]}_{s+-+-} = -\frac{1}{3}i\pi-\frac{5}{9} \tag{B.3}$$

$$A^{(1),[2]}_{s+-+-} = \Big(-\frac{13}{18}-\frac{1}{2}Y^2+\frac{3}{2}Y\Big)+i\pi\Big(-Y-\frac{1}{3}\Big) \tag{B.4}$$

$$\begin{aligned} B^{(1),[2]}_{s+-+-} &= \Big(-X+\frac{1}{4}X^2+\frac{3}{2}Y-\frac{1}{2}Y^2-4\Big)+\frac{1}{4}\frac{t^2}{u^2}X^2+\frac{1}{2}\frac{t}{u}X+ \\ &\quad +i\pi\Big(\frac{1}{2}\frac{t^2}{u^2}X+\frac{1}{2}\frac{t}{u}+\Big(\frac{1}{2}+\frac{1}{2}X-Y\Big)\Big) \end{aligned} \tag{B.5}$$

$$C^{(1),[2]}_{s+-+-} = \frac{5}{9}+\frac{1}{3}i\pi \tag{B.6}$$

B.2 $q(p_2,+)+\bar{q}(p_1,-)\to Q(p_3,-)+\bar{Q}(p_4,+)$

$$A^{(1),[1]}_{s+--+} = \Big(\frac{1}{2}X^2-\frac{3}{2}X+\frac{13}{18}\Big)+i\pi\Big(\frac{1}{3}+X\Big) \tag{B.7}$$

$$\begin{aligned} B^{(1),[1]}_{s+--+} &= \Big(3X+\frac{1}{2}Y^2-X^2-2Y+4\Big)+\frac{1}{2}\frac{u^2}{t^2}Y^2+\frac{u}{t}Y+ \\ &\quad +i\pi\Big(\frac{u^2}{t^2}Y+\frac{u}{t}+\Big(-2X+Y+1\Big)\Big) \end{aligned} \tag{B.8}$$

$$C^{(1),[1]}_{s+--+} = -\frac{1}{3}i\pi-\frac{5}{9} \tag{B.9}$$

$$\begin{aligned} A^{(1),[2]}_{s+--+} &= -\frac{1}{2}\frac{u}{t}Y-\frac{1}{4}\frac{u^2}{t^2}Y^2+\Big(-\frac{13}{18}-\frac{1}{4}Y^2+Y\Big) \\ &\quad +i\pi\Big(\Big(-\frac{1}{2}Y-\frac{5}{6}\Big)-\frac{1}{2}\frac{u^2}{t^2}Y-\frac{1}{2}\frac{u}{t}\Big) \end{aligned} \tag{B.10}$$

$$\begin{aligned} B^{(1),[2]}_{s+--+} &= -\frac{1}{2}\frac{u}{t}Y-\frac{1}{4}\frac{u^2}{t^2}Y^2+\Big(-\frac{3}{2}X+\frac{1}{2}X^2+Y-\frac{1}{4}Y^2-4\Big)+ \\ &\quad +i\pi\Big(-\frac{1}{2}\frac{u^2}{t^2}Y-\frac{1}{2}\frac{u}{t}+\Big(-\frac{1}{2}+X-\frac{1}{2}Y\Big)\Big) \end{aligned} \tag{B.11}$$

$$C^{(1),[2]}_{s+--+} = \frac{5}{9}+\frac{1}{3}i\pi \tag{B.12}$$

B.3 $q(p_2,+)+\bar{Q}(p_1,-)\to q(p_3,+)+\bar{Q}(p_4,-)$

$$\begin{aligned} A^{(1),[1]}_{t+-+-} &= \frac{t}{u}\Big(\frac{1}{2}X+\frac{1}{2}X^2\Big)+\frac{1}{4}\frac{t^2}{u^2}X^2+\Big(\frac{1}{2}X^2+\frac{13}{18}-\frac{1}{3}X\Big)+ \\ &\quad +i\pi\Big(\frac{1}{2}\frac{t^2}{u^2}X+\Big(X+\frac{3}{2}\Big)+\frac{t}{u}\Big(X+\frac{1}{2}\Big)\Big) \end{aligned} \tag{B.13}$$

$$\begin{aligned} B^{(1),[1]}_{t+-+-} &= -\frac{1}{2}\frac{t^2}{u^2}X^2+\frac{t}{u}\Big(-X-X^2\Big)+\Big(-2XY+Y^2+\pi^2+4-3Y\Big)+ \\ &\quad +i\pi\Big(-\frac{t^2}{u^2}X+\frac{t}{u}\Big(-2X-1\Big)+\Big(-2X-3\Big)\Big) \end{aligned} \tag{B.14}$$

$$C^{(1),[1]}_{t+-+-} = \left(-\frac{5}{9} + \frac{1}{3}X\right) \tag{B.15}$$

$$A^{(1),[2]}_{t+-+-} = \left(-\frac{1}{2}X^2 + \frac{3}{2}Y - \frac{1}{2}Y^2 + XY - \frac{1}{2}\pi^2 + \frac{1}{3}X - \frac{13}{18}\right) \tag{B.16}$$

$$\begin{aligned} B^{(1),[2]}_{t+-+-} = {} & \frac{1}{4}\frac{t^2}{u^2}X^2 + \frac{t}{u}\left(\frac{1}{2}X + \frac{1}{2}X^2\right) + \left(-\frac{1}{2}Y^2 + XY + \frac{3}{2}Y - 4 - \frac{1}{2}\pi^2\right) + \\ & + i\pi\left(\frac{1}{2}\frac{t^2}{u^2}X + \left(X + \frac{3}{2}\right) + \frac{t}{u}\left(X + \frac{1}{2}\right)\right) \end{aligned} \tag{B.17}$$

$$C^{(1),[2]}_{t+-+-} = \left(-\frac{1}{3}X + \frac{5}{9}\right) \tag{B.18}$$

B.4 $q(p_2,+) + \bar{Q}(p_1,+) \to q(p_3,+) + \bar{Q}(p_4,+)$

$$A^{(1),[1]}_{t++++} = \left(\frac{1}{2}X^2 + \frac{13}{18} - \frac{1}{3}X\right) + i\pi\left(X + \frac{3}{2}\right) \tag{B.19}$$

$$\begin{aligned} B^{(1),[1]}_{t++++} = {} & \frac{t^2}{s^2}\left(-XY + \frac{1}{2}X^2 + \frac{1}{2}Y^2 + \frac{1}{2}\pi^2\right) + \frac{t}{s}\left(-2XY + X + X^2 - Y + Y^2 + \pi^2\right) + \\ & + \left(-2XY + Y^2 + \pi^2 + 4 - 3Y\right) + i\pi\left(-2X - 3\right) \end{aligned} \tag{B.20}$$

$$C^{(1),[1]}_{t++++} = \left(-\frac{5}{9} + \frac{1}{3}X\right) \tag{B.21}$$

$$\begin{aligned} A^{(1),[2]}_{t++++} = {} & \frac{t}{s}\left(XY - \frac{1}{2}X - \frac{1}{2}X^2 + \frac{1}{2}Y - \frac{1}{2}Y^2 - \frac{1}{2}\pi^2\right) + \\ & + \frac{t^2}{s^2}\left(\frac{1}{2}XY - \frac{1}{4}X^2 - \frac{1}{4}Y^2 - \frac{1}{4}\pi^2\right) + \\ & + \left(-\frac{1}{2}X^2 + \frac{3}{2}Y - \frac{1}{2}Y^2 + XY - \frac{1}{2}\pi^2 + \frac{1}{3}X - \frac{13}{18}\right) \end{aligned} \tag{B.22}$$

$$\begin{aligned} B^{(1),[2]}_{t++++} = {} & \frac{t^2}{s^2}\left(\frac{1}{2}XY - \frac{1}{4}X^2 - \frac{1}{4}Y^2 - \frac{1}{4}\pi^2\right) + \\ & + \frac{t}{s}\left(XY - \frac{1}{2}X - \frac{1}{2}X^2 + \frac{1}{2}Y - \frac{1}{2}Y^2 - \frac{1}{2}\pi^2\right) + \\ & + \left(-\frac{1}{2}Y^2 + XY + \frac{3}{2}Y - 4 - \frac{1}{2}\pi^2\right) + i\pi\left(X + \frac{3}{2}\right) \end{aligned} \tag{B.23}$$

$$C^{(1),[2]}_{t++++} = \left(-\frac{1}{3}X + \frac{5}{9}\right) \tag{B.24}$$

B.5 $q(p_2,+) + Q(p_1,+) \to Q(p_3,+) + q(p_4,+)$

$$\begin{aligned} A^{(1),[1]}_{u++++} = {} & \frac{t}{s}\left(\frac{1}{2}X - \frac{1}{2}Y\right) + \frac{t^2}{s^2}\left(-\frac{1}{2}XY + \frac{1}{4}X^2 + \frac{1}{4}Y^2 + \frac{1}{4}\pi^2\right) + \\ & + \left(-\frac{1}{2}XY + \frac{1}{4}Y^2 - X + \frac{1}{4}\pi^2 + \frac{1}{4}X^2 + \frac{13}{18} - \frac{5}{6}Y\right) \end{aligned} \tag{B.25}$$

$$B^{(1),[1]}_{u++++} = \frac{t^2}{s^2}\left(XY - \frac{1}{2}X^2 - \frac{1}{2}Y^2 - \frac{1}{2}\pi^2\right) + \frac{t}{s}\left(-X + Y\right) + \\ + \left(XY + \frac{1}{2}Y^2 + 2X - \frac{1}{2}\pi^2 - \frac{1}{2}X^2 + 4 + Y\right) + +i\pi\left(2Y + 3\right) \qquad \text{(B.26)}$$

$$C^{(1),[1]}_{u++++} = \left(-\frac{5}{9} + \frac{1}{3}Y\right) \qquad \text{(B.27)}$$

$$A^{(1),[2]}_{u++++} = \left(-\frac{13}{18} - \frac{1}{2}Y^2 + \frac{1}{3}Y\right) + i\pi\left(-Y - \frac{3}{2}\right) \qquad \text{(B.28)}$$

$$B^{(1),[2]}_{u++++} = \frac{t^2}{s^2}\left(-\frac{1}{2}XY + \frac{1}{4}X^2 + \frac{1}{4}Y^2 + \frac{1}{4}\pi^2\right) + \frac{t}{s}\left(\frac{1}{2}X - \frac{1}{2}Y\right) + \\ + \left(-\frac{1}{2}Y - \frac{1}{2}XY - X + \frac{1}{4}X^2 + \frac{1}{4}\pi^2 - 4 - \frac{1}{4}Y^2\right) + i\pi\left(-Y - \frac{3}{2}\right) \qquad \text{(B.29)}$$

$$C^{(1),[2]}_{u++++} = \left(-\frac{1}{3}Y + \frac{5}{9}\right). \qquad \text{(B.30)}$$

B.6 $q(p_2,+) + Q(p_1,-) \to Q(p_3,-) + q(p_4,+)$

$$A^{(1),[1]}_{u+--+} = \left(-\frac{1}{3}Y + \frac{1}{2}Y^2 - XY + \frac{1}{2}\pi^2 - \frac{3}{2}X + \frac{13}{18} + \frac{1}{2}X^2\right) \qquad \text{(B.31)}$$

$$B^{(1),[1]}_{u+--+} = \frac{1}{2}\frac{u^2}{t^2}Y^2 + \frac{u}{t}\left(Y + Y^2\right) + \left(2XY - \pi^2 + 3X + 4 - X^2\right) + \\ + i\pi\left(\frac{u^2}{t^2}Y + \frac{u}{t}\left(2Y + 1\right) + \left(2Y + 3\right)\right) \qquad \text{(B.32)}$$

$$C^{(1),[1]}_{u+--+} = \left(-\frac{5}{9} + \frac{1}{3}Y\right) \qquad \text{(B.33)}$$

$$A^{(1),[2]}_{u+--+} = \frac{u}{t}\left(-\frac{1}{2}Y - \frac{1}{2}Y^2\right) - \frac{1}{4}\frac{u^2}{t^2}Y^2 + \left(-\frac{13}{18} - \frac{1}{2}Y^2 + \frac{1}{3}Y\right) + \\ + i\pi\left(-\frac{1}{2}\frac{u^2}{t^2}Y + \left(-Y - \frac{3}{2}\right) + \frac{u}{t}\left(-Y - \frac{1}{2}\right)\right) \qquad \text{(B.34)}$$

$$B^{(1),[2]}_{u+--+} = -\frac{1}{4}\frac{u^2}{t^2}Y^2 + \frac{u}{t}\left(-\frac{1}{2}Y - \frac{1}{2}Y^2\right) + \left(-4 - XY - \frac{3}{2}X + \frac{1}{2}X^2 + \frac{1}{2}\pi^2\right) + \\ + i\pi\left(-\frac{1}{2}\frac{u^2}{t^2}Y + \left(-Y - \frac{3}{2}\right) + \frac{u}{t}\left(-Y - \frac{1}{2}\right)\right) \qquad \text{(B.35)}$$

$$C^{(1),[2]}_{u+--+} = \left(-\frac{1}{3}Y + \frac{5}{9}\right). \qquad \text{(B.36)}$$

B.7 $q(p_2,+) + Q(p_1,+) \to q(p_3,+) + Q(p_4,+)$

The coefficients for this amplitude are obtained by exchanging t and u in the coefficients for $q(p_2,+) + Q(p_1,+) \to Q(p_3,+) + q(p_4,+)$,

$$A^{(1),[1]}_{t'++++} = A^{(1),[1]}_{u++++}(t \leftrightarrow u) \qquad \text{(B.37)}$$

$$A^{(1),[2]}_{t'++++} = A^{(1),[2]}_{u++++}(t \leftrightarrow u) \qquad \text{(B.38)}$$

B.8 $q(p_2,+) + Q(p_1,-) \to q(p_3,+) + Q(p_4,-)$

The coefficients for this amplitude are obtained by exchanging t and u in the coefficients for $q(p_2,+) + Q(p_1,-) \to Q(p_3,-) + q(p_4,+)$,

$$A^{(1),[1]}_{t'+-+-} = A^{(1),[1]}_{u+--+}(t \leftrightarrow u) \tag{B.39}$$

$$A^{(1),[2]}_{t'+-+-} = A^{(1),[2]}_{u+--+}(t \leftrightarrow u) \tag{B.40}$$

etc.

References

[1] C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *Two-loop QCD corrections to $q\bar{q} \to q'\bar{q}'$*, *Nucl. Phys.* **B 601** (2001) 318 [hep-ph/0010212].

[2] C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *Two-loop QCD corrections to $q\bar{q} \to q\bar{q}$*, *Nucl. Phys.* **B 601** (2001) 341 [hep-ph/0011094].

[3] E.W.N. Glover, *Progress in NNLO calculations for scattering processes*, *Nucl. Phys.* **116** *(Proc. Suppl.)* (2003) 3 [hep-ph/0211412].

[4] S.A. Larin, P. Nogueira, T. van Ritbergen and J.A.M. Vermaseren, *The 3-loop QCD calculation of the moments of deep inelastic structure functions*, *Nucl. Phys.* **B 492** (1997) 338 [hep-ph/9605317].

[5] S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, *The next next-to-leading QCD approximation for nonsinglet moments of deep inelastic structure functions*, *Nucl. Phys.* **B 427** (1994) 41.

[6] A. Retey and J.A.M. Vermaseren, *Some higher moments of deep inelastic structure functions at next-to-next-to leading order of perturbative QCD*, *Nucl. Phys.* **B 604** (2001) 281 [hep-ph/0007294].

[7] J.A. Gracey, *Anomalous dimension of nonsinglet Wilson operators at $O(1/N_f)$ in deep inelastic scattering*, *Phys. Lett.* **B 322** (1994) 141 [hep-ph/9401214].

[8] W.L. van Neerven and A. Vogt, *NNLO evolution of deep-inelastic structure functions: the non-singlet case*, *Nucl. Phys.* **B 568** (2000) 263 [hep-ph/9907472].

[9] W.L. van Neerven and A. Vogt, *NNLO evolution of deep-inelastic structure functions: the singlet case*, *Nucl. Phys.* **B 588** (2000) 345 [hep-ph/0006154].

[10] W.L. van Neerven and A. Vogt, *Improved approximations for the three-loop splitting functions in QCD*, *Phys. Lett.* **B 490** (2000) 111 [hep-ph/0007362].

[11] J.A.M. Vermaseren, S. Moch and A. Vogt, *First results for three-loop deep-inelastic structure functions in QCD*, *Nucl. Phys.* **116** *(Proc. Suppl.)* (2003) 100 [hep-ph/0211296].

[12] C.F. Berger, *Higher orders in $A(\alpha_s)/[1-x]_+$ of non-singlet partonic splitting functions*, *Phys. Rev.* **D 66** (2002) 116002 [hep-ph/0209107].

[13] D.A. Kosower and P. Uwer, *One-loop splitting amplitudes in gauge theory*, *Nucl. Phys.* **B 563** (1999) 477 [hep-ph/9903515].

[14] D.A. Kosower and P. Uwer, *Evolution kernels from splitting amplitudes*, *Nucl. Phys.* **B 674** (2003) 365 [hep-ph/0307031].

JHEP04(2004)021

[15] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, *Estimating the effect of NNLO contributions on global parton analyses*, *Eur. Phys. J.* **C 18** (2000) 117 [hep-ph/0007099].

[16] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, *NNLO global parton analysis*, *Phys. Lett.* **B 531** (2002) 216 [hep-ph/0201127].

[17] A. Vogt, S. Moch and J.A.M. Vermaseren, *The three-loop splitting functions in QCD: the singlet case*, hep-ph/0404111.

[18] A. Vogt, S. Moch and J.A.M. Vermaseren, *The three-loop splitting functions in QCD: the non-singlet case*, hep-ph/0403192.

[19] J.F. Gunion and J. Kalinowski, *A direct calculation of the six gluon subprocess*, *Phys. Rev.* **D 34** (1986) 2119.

[20] S.J. Parke and T.R. Taylor, *Gluonic two goes to four*, *Nucl. Phys.* **B 269** (1986) 410.

[21] F.A. Berends and W. Giele, *The six gluon process as an example of Weyl-Van der Waerden spinor calculus*, *Nucl. Phys.* **B 294** (1987) 700.

[22] M.L. Mangano, S.J. Parke and Z. Xu, *Duality and multi-gluon scattering*, *Nucl. Phys.* **B 298** (1988) 653.

[23] Z. Kunszt, *Combined use of the calkul method and $N = 1$ supersymmetry to calculate QCD six parton processes*, *Nucl. Phys.* **B 271** (1986) 333.

[24] S.J. Parke and T.R. Taylor, *The cross-section for hard processes involving two quarks and four gluons*, *Phys. Rev.* **D 35** (1987) 313.

[25] J.F. Gunion and Z. Kunszt, *Four jet processes: gluon-gluon scattering to nonidentical quark–anti-quark pairs*, *Phys. Lett.* **B 159** (1985) 167.

[26] J.F. Gunion and Z. Kunszt, *Six quark subprocesses in QCD*, *Phys. Lett.* **B 176** (1986) 163.

[27] Z. Bern, L.J. Dixon and D.A. Kosower, *One loop corrections to five gluon amplitudes*, *Phys. Rev. Lett.* **70** (1993) 2677 [hep-ph/9302280].

[28] Z. Bern, L.J. Dixon and D.A. Kosower, *One loop corrections to two quark three gluon amplitudes*, *Nucl. Phys.* **B 437** (1995) 259 [hep-ph/9409393].

[29] Z. Kunszt, A. Signer and Z. Trócsányi, *One loop radiative corrections to the helicity amplitudes of QCD processes involving four quarks and one gluon*, *Phys. Lett.* **B 336** (1994) 529 [hep-ph/9405386].

[30] C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *Two-loop QCD corrections to massless quark gluon scattering*, *Nucl. Phys.* **B 605** (2001) 486 [hep-ph/0101304].

[31] E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *Two-loop QCD corrections to gluon gluon scattering*, *Nucl. Phys.* **B 605** (2001) 467 [hep-ph/0102201].

[32] C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *One-loop QCD corrections to quark scattering at NNLO*, *Phys. Lett.* **B 506** (2001) 59 [hep-ph/0012007].

[33] E.W.N. Glover and M.E. Tejeda-Yeomans, *One-loop QCD corrections to gluon gluon scattering at NNLO*, *J. High Energy Phys.* **05** (2001) 010 [hep-ph/0104178].

[34] Z. Bern, L.J. Dixon and D.A. Kosower, *A two-loop four-gluon helicity amplitude in QCD*, *J. High Energy Phys.* **01** (2000) 027 [hep-ph/0001001].

[35] Z. Bern, A. De Freitas and L.J. Dixon, *Two-loop helicity amplitudes for gluon gluon scattering in QCD and supersymmetric Yang-Mills theory*, *J. High Energy Phys.* **03** (2002) 018 [hep-ph/0201161].

[36] Z. Bern, A. De Freitas and L.J. Dixon, *Two-loop helicity amplitudes for quark gluon scattering in QCD and gluino gluon scattering in supersymmetric Yang-Mills theory*, *J. High Energy Phys.* **06** (2003) 028 [hep-ph/0304168].

[37] E.W.N. Glover and M.E. Tejeda-Yeomans, *Two-loop QCD helicity amplitudes for massless quark massless gauge boson scattering*, *J. High Energy Phys.* **06** (2003) 033 [hep-ph/0304169].

[38] Z. Bern, A. De Freitas and L.J. Dixon, *Two-loop amplitudes for gluon fusion into two photons*, *J. High Energy Phys.* **09** (2001) 037 [hep-ph/0109078].

[39] L.W. Garland, T. Gehrmann, E.W.N. Glover, A. Koukoutsakis and E. Remiddi, *Two-loop QCD helicity amplitudes for $e^+e^- \to$ 3jets*, *Nucl. Phys.* **B 642** (2002) 227 [hep-ph/0206067].

[40] Z. Bern, A. De Freitas, L.J. Dixon, A. Ghinculov and H.L. Wong, *QCD and QED corrections to light-by-light scattering*, *J. High Energy Phys.* **11** (2001) 031 [hep-ph/0109079].

[41] T. Binoth, E.W.N. Glover, P. Marquard and J.J. van der Bij, *Two-loop corrections to light-by-light scattering in supersymmetric QED*, *J. High Energy Phys.* **05** (2002) 060 [hep-ph/0202266].

[42] S. Catani, *The singular behaviour of QCD amplitudes at two-loop order*, *Phys. Lett.* **B 427** (1998) 161 [hep-ph/9802439].

[43] G. 't Hooft and M.J.G. Veltman, *Regularization and renormalization of gauge fields*, *Nucl. Phys.* **B 44** (1972) 189.

[44] G. 't Hooft, *Dimensional regularization and the renormalization group*, *Nucl. Phys.* **B 61** (1973) 455.

[45] C.G. Bollini and J.J. Giambiagi, *Dimensional renormalization: the number of dimensions as a regularizing parameter*, *Nuovo Cim.* **B12** (1972) 20.

[46] G.M. Cicuta and E. Montaldi, *Analytic renormalization via continuous space dimension*, *Lett. Nuovo Cim.* **4** (1972) 329.

[47] R.K. Ellis and J.C. Sexton, *QCD radiative corrections to parton parton scattering*, *Nucl. Phys.* **B 269** (1986) 445.

[48] P. Nogueira, *Automatic feynman graph generation*, *J. Comput. Phys.* **105** (1993) 279.

[49] MAPLE V *release 7*, by Waterloo Maple Inc. Copyright 2001.

[50] J.A.M. Vermaseren, *New features of form*, math-ph/0010025.

[51] W. Giele et al., *The QCD/SM working group: summary report*, hep-ph/0204316.

[52] T. Gehrmann, *Jet physics at two-loop accuracy*, *Nucl. Phys.* **116** *(Proc. Suppl.)* (2003) 13 [hep-ph/0210157].

[53] T. Gehrmann, *Theoretical developments on hard QCD processes at colliders*, hep-ph/0211169.

[54] S. Laporta, *High-precision calculation of multi-loop Feynman integrals by difference equations*, *Int. J. Mod. Phys.* **A 15** (2000) 5087 [hep-ph/0102033].

JHEP04(2004)021

[55] K.G. Chetyrkin, A.L. Kataev and F.V. Tkachov, *New approach to evaluation of multiloop Feynman integrals: the Gegenbauer polynomial x space technique*, *Nucl. Phys.* **B 174** (1980) 345.

[56] K.G. Chetyrkin and F.V. Tkachov, *Integration by parts: the algorithm to calculate beta functions in 4 loops*, *Nucl. Phys.* **B 192** (1981) 159.

[57] T. Gehrmann and E. Remiddi, *Differential equations for two-loop four-point functions*, *Nucl. Phys.* **B 580** (2000) 485 [hep-ph/9912329].

[58] R.J. Gonsalves, *Dimensionally regularized two loop on-shell quark form-factor*, *Phys. Rev.* **D 28** (1983) 1542.

[59] G. Kramer and B. Lampe, *Integrals for two loop calculations in massless QCD*, *J. Math. Phys.* **28** (1987) 945.

[60] C. Anastasiou, E.W.N. Glover and C. Oleari, *Scalar one-loop integrals using the negative-dimension approach*, *Nucl. Phys.* **B 572** (2000) 307 [hep-ph/9907494].

[61] C. Anastasiou, E.W.N. Glover and C. Oleari, *Application of the negative-dimension approach to massless scalar box integrals*, *Nucl. Phys.* **B 565** (2000) 445 [hep-ph/9907523].

[62] C. Anastasiou, E.W.N. Glover and C. Oleari, *The two-loop scalar and tensor pentabox graph with light-like legs*, *Nucl. Phys.* **B 575** (2000) 416 [hep-ph/9912251], erratum *ibid.* **B585** (2000) 763.

[63] V.A. Smirnov, *Analytical result for dimensionally regularized massless on-shell double box*, *Phys. Lett.* **B 460** (1999) 397 [hep-ph/9905323].

[64] J.B. Tausk, *Non-planar massless two-loop Feynman diagrams with four on-shell legs*, *Phys. Lett.* **B 469** (1999) 225 [hep-ph/9909506].

[65] V.A. Smirnov and O.L. Veretin, *Analytical results for dimensionally regularized massless on-shell double boxes with arbitrary indices and numerators*, *Nucl. Phys.* **B 566** (2000) 469 [hep-ph/9907385].

[66] C. Anastasiou, T. Gehrmann, C. Oleari, E. Remiddi and J.B. Tausk, *The tensor reduction and master integrals of the two-loop massless crossed box with light-like legs*, *Nucl. Phys.* **B 580** (2000) 577 [hep-ph/0003261].

[67] C. Anastasiou, J.B. Tausk and M.E. Tejeda-Yeomans, *The on-shell massless planar double box diagram with an irreducible numerator*, *Nucl. Phys.* **89** *(Proc. Suppl.)* (2000) 262 [hep-ph/0005328].

[68] G. Sterman and M.E. Tejeda-Yeomans, *Multi-loop amplitudes and resummation*, *Phys. Lett.* **B 552** (2003) 48 [hep-ph/0210130].

[69] Z. Bern, L.J. Dixon and A. Ghinculov, *Two-loop correction to Bhabha scattering*, *Phys. Rev.* **D 63** (2001) 053007 [hep-ph/0010075].

[70] C. Anastasiou, E.W.N. Glover and M.E. Tejeda-Yeomans, *Two-loop qed and QCD corrections to massless fermion boson scattering*, *Nucl. Phys.* **B 629** (2002) 255 [hep-ph/0201274].

[71] S. Catani and M. Grazzini, *The soft-gluon current at one-loop order*, *Nucl. Phys.* **B 591** (2000) 435 [hep-ph/0007142].

[72] N. Kidonakis, G. Oderda and G. Sterman, *Evolution of color exchange in QCD hard scattering*, *Nucl. Phys.* **B 531** (1998) 365 [hep-ph/9803241].

[73] Z. Kunszt, A. Signer and Z. Trócsányi, *One loop helicity amplitudes for all $2 \to 2$ processes in QCD and $N = 1$ supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 411** (1994) 397 [hep-ph/9305239].

[74] M.E. Tejeda-Yeomans, *Parton-parton scattering at two-loops*, Ph.D. thesis, University of Durham, 2001.

[75] W.T. Giele, E.W.N. Glover and D.A. Kosower, *Higher order corrections to jet cross-sections in hadron colliders*, *Nucl. Phys.* **B 403** (1993) 633 [hep-ph/9302225].

[76] W.T. Giele, E.W.N. Glover and D.A. Kosower, *The inclusive two jet triply differential cross-section*, *Phys. Rev.* **D 52** (1995) 1486 [hep-ph/9412338].

[77] S.D. Ellis, Z. Kunszt and D.E. Soper, *The one jet inclusive cross-section at order α_s^3 quarks and gluons*, *Phys. Rev. Lett.* **64** (1990) 2121.

[78] S.D. Ellis, Z. Kunszt and D.E. Soper, *Two jet production in hadron collisions at order α_s^3 in QCD*, *Phys. Rev. Lett.* **69** (1992) 1496.

[79] D.A. Kosower, *Multiple singular emission in gauge theories*, *Phys. Rev.* **D 67** (2003) 116003 [hep-ph/0212097].

[80] D.A. Kosower, *All-orders singular emission in gauge theories*, *Phys. Rev. Lett.* **91** (2003) 061602 [hep-ph/0301069].

[81] D.A. Kosower, *Antenna factorization in strongly-ordered limits*, hep-ph/0311272.

[82] S. Weinzierl, *Subtraction terms at NNLO*, *J. High Energy Phys.* **03** (2003) 062 [hep-ph/0302180].

[83] A. Gehrmann-De Ridder, T. Gehrmann and G. Heinrich, *Four-particle phase space integrals in massless QCD*, hep-ph/0311276.

[84] C. Anastasiou, K. Melnikov and F. Petriello, *A new method for real radiation at NNLO*, hep-ph/0311311.

[85] Z. Bern, A. De Freitas and L. Dixon, work in preparation.

[86] K.S. Kolbig, J.A. Mignaco, and E. Remiddi, *On Nielsen's generalised polylogarithms and their numerical calculation*, *B.I.T.* **10** (1970) 38.